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THESIS

**DIGITAL ANTENNA ARCHITECTURES USING
COMMERCIAL OFF THE SHELF HARDWARE**

by

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**DIGITAL ANTENNA ARCHITECTURES
USING COMMERCIAL OFF-THE-SHELF HARDWARE**

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ABSTRACT

The changes in war fighting tactics and advancement of technology shape the ways to implement and design multifunction phased array radars. This thesis investigated whether the commercial modulation boards used in the 3-D 2.4-GHz phased array transmit antenna is capable of wideband performance. The phase of the transmitted signal out of the modulator board was adjusted to provide a phase shift from 0 to 2π , and the insertion phases at these phase settings were measured using a Vector Network Analyzer, sweeping the frequency from 0.8 to 2.5 GHz. The measured insertion phases are used to simulate the radiation pattern of a linear phased array, and the results show that the modulator does not have instantaneous wideband characteristics.

This thesis also looked at the design of the complementary phased array receiver architecture using commercially available demodulator boards. The demodulator board was successfully configured to operate as a phase shifter. Phase shifted transmit signals were injected into the demodulator, and corresponding phase shifts were measured via the *In-Phase* and *Quadrature* voltages.

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TABLE OF CONTENTS

I.	INTRODUCTION.....	1
A.	MOTIVATION	1
B.	PREVIOUS WORK.....	4
C.	SCOPE AND ORGANIZATION	4
1.	Scope.....	4
2.	Primary Research Questions	5
3.	Organization.....	5
II.	PHASED ARRAYS.....	7
A.	ARRAYS.....	7
B.	ARRAY FACTOR (AF)	7
C.	ARRAY BANDWIDTH	9
D.	ADAPTIVE BEAMFORMING.....	11
E.	DIRECTION FINDING	12
F.	<i>I</i> AND <i>Q</i> AND DEMODULATION SCHEME	15
G.	DIGITAL ANTENNA ARCHITECTURE.....	16
H.	SUMMARY	17
III.	DESIGN SELECTION AND MEASUREMENT ANALYSIS	19
A.	THREE-DIMENSIONAL 2.4-GHZ PHASED ARRAY TRANSMIT ANTENNA.....	19
1.	AD8346EVAL Quadrature Modulator.....	19
2.	Operation of the AD8346EVAL Board as a Phase Shifter	20
3.	Magnitude and Phase Measurements for the AD8346EVAL Board	21
4.	Implementation of a Linear Array.....	30
a.	<i>Parameter Specifications</i>	30
b.	<i>Phase Reference and Phase Shifts</i>	31
c.	<i>Radiation Patterns</i>	33
B.	PHASED ARRAY RECEIVER.....	37
1.	Receiver Component Design Considerations	37
a.	<i>Option One</i>	37
b.	<i>Option Two</i>	38
c.	<i>Option Three</i>	38
2.	AD8347EVAL Quadrature Demodulator.....	39
3.	AD8347EVAL Quadrature Demodulator Bench Test Configuration	41
4.	Experimental Measurements	43
C.	SUMMARY	49
IV.	CONCLUSIONS AND RECOMMENDATIONS.....	51
A.	SUMMARY	51
B.	RECOMMENDATIONS FOR FUTURE WORK.....	51

1.	Arrays Transmit Antenna with Time Delay Unit	51
2.	Transmit Antenna with Amplitude Equalizers	52
3.	Receive Antenna.....	52
APPENDIX A: GLOSSARY OF TERMINOLOGY		53
APPENDIX B: MATLAB CODE OF THE PHASED ARRAY TRANSMIT ANTENNA FOR GENERATING FIGURES 17 TO 20		55
APPENDIX C: MATHCAD CODE FOR SIMULATING THE RADIATION PATTERN OF THE LINEAR PHASED ARRAY		59
APPENDIX D: MATLAB CODE FOR ARRAY RECEIVER.....		73
LIST OF REFERENCES		75
INITIAL DISTRIBUTION LIST		77

LIST OF FIGURES

Figure 1.	SPY-1 Antenna Face (From Ref. [1].)	2
Figure 2.	Subarray Antennas in the DD(X) Superstructure (From Ref. [2].)	3
Figure 3.	Different Array Geometries for Phased Array Antennas (From Ref. [6].)	8
Figure 4.	Geometry of the Uniformly Spaced Linear Array.	8
Figure 5.	A Sample Array Pattern (From Ref. [11].)	11
Figure 6.	Two-Element Linear Array.....	12
Figure 7.	Receiver Circuit for an Element of the Array.....	14
Figure 8.	Architecture of a Digital Phased Array Transmit Antenna.....	16
Figure 9.	Architecture of a Digital Phased Array Receive Antenna.	17
Figure 10.	Complex (Phasor) Plane (From Ref. [3].).....	20
Figure 11.	AD8346EVAL Bench Test Configuration.....	21
Figure 12.	AD8346EVAL Cable and Signal Connections (From Ref. [3].)	22
Figure 13.	The Control Program GUI.....	22
Figure 14.	Radar Equipment Cart (From Ref. [3].).....	23
Figure 15.	Frequency-Magnitude Response for Zero Phase Setting.....	24
Figure 16.	Frequency-Phase Response for Zero Phase setting.	24
Figure 17.	Magnitude and Phase Versus Frequency Response for Phase Shift From 0° to 75° at 5° Increments.....	26
Figure 18.	Magnitude and Phase Versus Frequency Response for Phase Shift From 90° to 165° at 5° Increments. (The 0° Phase Shift is Plotted for Reference.)	27
Figure 19.	Magnitude and Phase Versus Frequency Response for Phase Shift From 180° to 255° at 5° Increments. (The 0° Phase Shift is Plotted for Reference.)	28
Figure 20.	Magnitude and Phase Versus Frequency Response for Phase Shift From 270° to 345° at 5° Increments. (The 0° Phase Shift is Plotted for Reference.)	29
Figure 21.	Thirty-Element Linear Array.....	31
Figure 22.	Frequency-Phase Response for a 93° Phase Shift Setting.....	33
Figure 23.	Radiation Pattern at $f_{oper} = 1.7$ GHz	34
Figure 24.	Radiation Pattern at $f_{oper} = 0.8$ GHz Compared to $f_{ref} = 1.7$ GHz.	35
Figure 25.	Radiation Pattern at $f_{oper} = 2.5$ GHz Compared to $f_{ref} = 1.7$ GHz.	36
Figure 26.	Option One for Array Receiver.....	37
Figure 27.	Option Two for Array Antenna.....	38
Figure 28.	Option Three for Array Antenna.....	39
Figure 29.	AD8347EVAL Circuit Diagram (From Ref. [17].)	40
Figure 30.	AD8347EVAL Bench Test Configuration.....	41
Figure 31.	AD8347EVAL Cable/Signal Connections.	42
Figure 32.	Measured Differential I and Q Components Without and With Offsets.	47

Figure 33.	Measured Differential <i>I</i> and <i>Q</i> Components Versus Transmitted Phase Response.....	48
Figure 34.	Transmitted Phase Versus Measured Phase Response.	49

LIST OF TABLES

Table 1.	Control Signal Value for Phase Shift (From Ref. [3].).....	21
Table 2.	Phase Shift Associated With Each Element.....	31
Table 3.	Results of AD8347EVAL Bench Test.....	43

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EXECUTIVE SUMMARY

Operation in a littoral environment calls for a ship self-defense system capable of coping with a high intensity, combined missile, aircraft and surface threats, under severe jamming and clutter conditions. One of the important components of the ship self-defense philosophy is stealth technology, or signature control.

In addition, operation in a littoral environment demands that the radar system provide long range detection and accurate tracking, and this implies physically large radar antennas with high gain and narrow beamwidth. However, this requirement conflicts with the stealth technology requirement, putting constraints on the design of the warship.

A possible solution to the conflicting requirements is to design a warship's structure to possess all the necessary stealth characteristics, and then integrate into the ship's structure several smaller noncontiguous subarray antennas that process the received signal coherently. These subarray elements not only serve as an aperture for radar applications, but also serve as apertures for a wide spectrum communication applications.

Unlike the military hardware of the past, information technology is advancing at a pace driven not by the military's requirements, but by the industrial and consumer sectors. These phenomena are mostly due to the development of Commercial-off-the-Shelf (COTS) microchips. Number-crunching chips are becoming more powerful while their cost and physical size are shrinking. Thus engineers should exploit commercial technologies for radar and communications applications.

Digital antennas have the potential to satisfy these diverse requirements simultaneously. The ideal digital antenna has the following features:

1. Potential ultra-wideband performance,
2. Use of distributed noncontiguous areas of the ship or aircraft for the antenna aperture,

3. Ability to serve several systems and perform multiple functions simultaneously (e.g., multiple beams, direction finding, search and track, and communications),
4. Low cost through the implementation of COTS,
5. Flexibility to change system parameters with simple software modifications.

This thesis is the continuation of research on a 3-D 2.4-GHz digital phased array radar antenna carried out at the Naval Postgraduate School. This thesis investigated whether the commercial modulator board used in the phased array transmit antenna is capable of wideband performance. The phase of the transmitted signal out of the modulator board was adjusted to provide a phase shift from 0 to 2π , and the insertion phases at these phase settings were measured using a Vector Network Analyzer, sweeping the frequency from 0.8 to 2.5 GHz. The effects of the phase slope and amplitude variations on the pattern of a linear array were determined by simulations that incorporated the measured data. The simulation showed that the instantaneous bandwidth of the modulator board is much narrower than the specified frequency range of operation (0.8 to 2.5 GHz). Thus, this modulator board does not have wide instantaneous bandwidth characteristics.

This thesis also looked at the design of the complementary phased array receiver architecture using commercial Radio Frequency (RF) microchips. The operating characteristics of an available quadrature demodulator were studied, and the results are presented. The demodulator was successfully configured to operate as a phase shifter. Phase shifted transmit signals from the modulator board were injected into the demodulator, and corresponding phase shifts were measured via the *In-phase* and *Quadrature* voltages. A periodic phase error was identified and attributed to the phase error inherent in the modulator board.

I. INTRODUCTION

A. MOTIVATION

Radar continues to be the principal above-water sensor for surface combatants. However, changes in the global security situation have led to navies sending their warships to operate much closer to potentially hostile coasts, e.g., in the Persian Gulf, as opposed to traditional operations in the open ocean. The operation in a littoral environment calls for a ship self-defense system capable of coping with a high intensity, combined missile, aircraft and surface threats, under severe jamming and clutter conditions.

One of the important components of the ship self defense philosophy is stealth technology, or signature control. Warships are now being designed for minimal platform signatures in the Radar Cross Section (RCS), Infrared (IR), magnetic, and acoustic domains. The design of minimal RCS and IR signatures, on one hand, calls for a small-integrated superstructure of the warship, which implies reduced area for sensors and weapon systems. On the other hand, the threats in the littoral environment require the radar to provide long-range detection and accurate tracking, and this implies physically large radar antennas with high gain and narrow beamwidth. Current radar array antennas are large planar arrays and easily recognizable, e.g., the SPY-1 radar used in the AEGIS combat systems, shown in Figure 1. These array antennas and their support structures are a primary driving factor in the design and construction of a warship.



Figure 1. SPY-1 Antenna Face (From Ref. [1].).

A possible solution to the conflicting design requirements is to define the warship's structure that possesses all of the necessary stealth characteristics, as well as maximized survivability and maneuverability, and then to integrate into the ship's structure several smaller noncontiguous subarray antennas that process the received signal coherently. A radar designed in this way would have good angular resolution since the angular resolution (i.e., half power beamwidth, θ_B) is proportional to the ratio of wavelength over the aperture width of the array as given by

$$\theta_B = 65 \frac{\lambda}{D}, \quad (1)$$

where λ is the wavelength and D is the dimension of the antenna. Thus, the aperture width could potentially be the overall length of the warship. These integrated ship-wide subarray antennas are evident in the proposed new DD(X) class destroyer design, as illustrated in Figure 2. These array elements not only serve as an aperture for radar applications, but also serve as apertures for a wide spectrum of communication applications. A single antenna with wideband performance can eliminate the numerous antennas and masts populating the current ship superstructure. In addition, operating in the UHF/VHF frequency band has the advantages of longer range detection and anti-stealth.

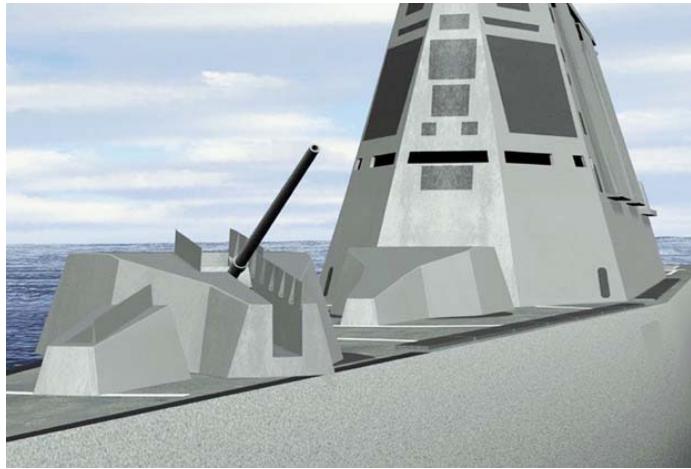


Figure 2. Subarray Antennas in the DD(X) Superstructure (From Ref. [2].).

Such a distributed subarray is possible through the use of a branch of computer programs known as Genetic Algorithms (GAs). The GA can determine the evolved array configuration that is compatible with the naval architecture's design criteria, instead of restricting the ship design due to radar location [3]. A more detailed description of the GA can be found in [3].

Today's naval radars have double the performance of those of the early nineteen eighty's, yet they cost the same [4]. In addition, unlike the military hardware of the past, information technology is advancing at a pace driven not by the military's requirements, but by the industrial and consumer sectors. These phenomena are mostly due to the development of Commercial-off-the-Shelf (COTS) microchips. Number-crunching chips are becoming more powerful while their cost and physical size are shrinking. Hence engineers have to leverage and evaluate commercial technologies for radar and communications architectures now and in the future.

Digital antennas have the potential to satisfy these diverse requirements simultaneously. The ideal digital antenna has the following features:

1. Potential ultra-wideband performance,
2. Use of distributed noncontiguous areas of the ship or aircraft for antenna aperture,

3. Ability to serve several systems and perform multiple functions simultaneously (e.g., multiple beams, direction finding, search and track, and communications),
4. Low cost through the implementation of COTS,
5. Flexibility to change system parameters with simple software modifications.

B. PREVIOUS WORK

This thesis is the continuation of the efforts to achieve a 3-D 2.4-GHz digital phased array radar. The transmit antenna design was carried out at the Naval Postgraduate School by LCDR Lance C. Esswein, USN [3]. Esswein had designed a phased array transmit antenna using COTS products and demonstrated that the GA program and its pattern builder function would form a radiation beam in agreement with the theoretical calculations.

The Analog Devices (AD) AD8346EVAL modulator boards were assembled into a twenty-four element array. These modulators boards were configured as phase shifters, each feeding one of twenty-four printed circuit dipoles acting as the radiating elements. Esswein verified that the radiation pattern measurements taken in the anechoic chamber were in good agreement with those predicted by numerical simulation.

C. SCOPE AND ORGANIZATION

1. Scope

In [3], only phase scanning at a single frequency of 2.4 GHz was used in the demonstration. As a continuation of the research, this thesis investigated whether the modulator board used in the phased array transmit antenna is capable of wideband performance. The phase of the transmitted signal out of the modulator board was adjusted to provide a phase shift from 0 to 2π , and the insertion phases at these phase settings were measured using a Vector Network Analyzer, sweeping the frequency from 0.8 to 2.5 GHz. The measured insertion phases were used to simulate the radiation pattern of a linear phased array.

This thesis also looked at the design of the complementary phased array receiver architecture using commercial Radio Frequency (RF) microchips. The operating characteristics of an available quadrature demodulator were studied, and the results are presented.

2. Primary Research Questions

There are two related research questions addressed in this thesis. The first was to examine the operating characteristics of the COTS modulator and demodulator for digital phased array applications. The second was to access possible digital antenna architectures that might use the COTS products.

3. Organization

Chapter II presents a brief overview of the phased array design addressing the calculation of the Array Factor (AF), wideband array characteristics and adaptive beamforming. The processes of direction finding and receiver demodulation are also discussed.

Chapter III provides the analysis of the measured scattering parameters (S_{21}) for the phased array transmit antenna in terms of amplitude and phase. In addition, these measured data are used to verify whether the modulator board has wideband performance by simulating radiation patterns of a linear phased array. The demodulator board for the phased array receiver is characterized and the results are presented.

Chapter IV contains the summary of the experimental results followed by suggestions for future research into the phased array architecture using COTS.

Appendix A is the glossary of terms and abbreviations used within this thesis. Appendix B is the MATLAB code for generating Figures 17 to 20. Appendix C is the MathCAD codes for simulating the radiation pattern of the linear phased array. Appendix D is the MATLAB code for the phased array receiver.

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II. PHASED ARRAYS

This chapter discusses the fundamental concept and theory of phased arrays such as array factor, bandwidth, and adaptive beamforming. The processes of direction finding and receiver demodulation are also discussed.

A. ARRAYS

A single element has relatively wide radiation pattern and it has low values of directivity and gain [5]. In many applications, especially radar defense systems, the antenna must have very directive characteristics and high gain to meet the demand of high resolution in both angle and range. This can be realized by two methods. The first method is to increase the electrical size of the antenna. The second is to form an assembly of radiating elements in an electrical and geometrical configuration without increasing the dimension of the individual elements. This second approach, formed by multi-elements, is referred to as an *array*.

Arrays offer the unique capability to steer the radar beam virtually instantaneously, without mechanical movement, in any required direction by simply changing the phase of the exciting currents in each element of the array. Such an array is called a *phased array*.

B. ARRAY FACTOR (AF)

A phased array can be geometrically constructed around a circle (circular array), along a line (linear array), on plane (planar array), or throughout a volume, as shown in Figure 3.

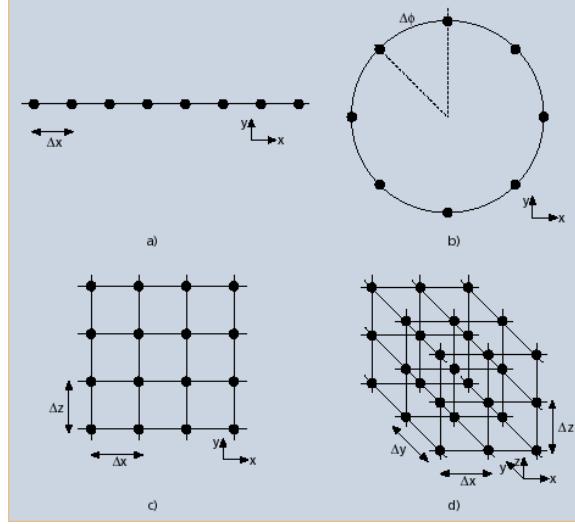


Figure 3. Different Array Geometries for Phased Array Antennas (From Ref. [6].).

For simplicity, the linear array is used to illustrate the concept of array factor. The array of Figure 4 has uniformly spaced identical isotropic radiators, positioned along the x -axis. The pattern characteristic of the array can be explained in either the transmit or receive mode, since the antenna usually satisfies the conditions for reciprocity [7].

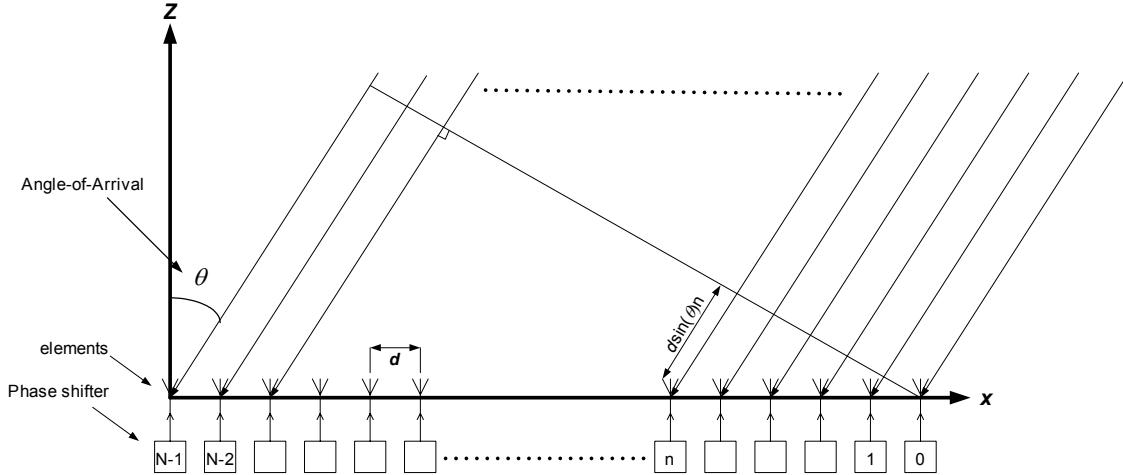


Figure 4. Geometry of the Uniformly Spaced Linear Array.

The parameter d represents the separation between antenna elements. Because the wave is incident at an angle θ relative to the axis of the array, all of the elements will receive the signal at different time, and thus be different in phase angle (i.e., phase shifted). We can determine the signal angle-of-arrival by measuring the phase difference

between the elements and then sum up all of the signals received by individual elements in order to maximize the received signal. Therefore by controlling the delays (phase shifts), we can select the direction of maximum reception or transmission for the array.

The array factor, AF for an N element uniform linear array along the x -axis is given by [5]

$$AF(\theta) = \sum_{n=0}^{N-1} A_n e^{j\Phi_n} e^{-j\frac{2\pi}{\lambda}nd \sin(\theta)}, \quad (2)$$

where

N = number of elements,

$A_n e^{j\Phi_n}$ = complex coefficient of element n used to steer the beam
in the desired direction,

$$= A_n e^{j\frac{2\pi}{\lambda}nd \sin(\theta_s)},$$

θ_s = scanning direction of the main beam,

d = spacing between elements in terms of wavelength,

λ = wavelength, and

A_n = magnitude of voltage or current at element n (assume all $A_n = 1$).

The far-field pattern of Equation (2) assumes that all the elements are identical and uniformly spaced. Isotropic elements have been assumed. If the elements are not isotropic, then the total radiation pattern of the array is the product of the element pattern and the array factor.

C. ARRAY BANDWIDTH

Equation (2) is valid for a single wavelength λ , i.e., narrowband, but in the wideband case, λ will cover a range of wavelengths. Hence Equation (2) is modified to accommodate the array pattern as a function of radiating frequency and angle. Substitute $\lambda = c/f$ in Equation (2) to obtain

$$AF(f, \theta) = \sum_{n=0}^{N-1} A_n(f) e^{j\frac{2\pi f}{c}nd \sin(\theta_s)} e^{-j\frac{2\pi f}{c}nd \sin(\theta)}, \quad (3)$$

where

$$\Phi_n(f) = \frac{2\pi f}{c}nd \sin(\theta_s),$$

$A_n(f)$ = a set of real weights, and

c = speed of light.

If the frequency (or wavelength) of operation changes, then a new set of phase shift settings is required to keep the beam location fixed. That is, the first exponent $\Phi_n(f)$ in Equation (3) must change with frequency in the same way that the second exponent changes with frequency. This would be a true time delay (TTD) phase shift. It will be shown that the COTS boards do not provide TTD. The phase shifter setting for a board is fixed for all frequencies. When the operating frequency changes, then the main beam will steer off from the desired direction, changing both in width and direction with frequency [8]. This effect is sometimes referred to as beam squint.

The beam squint is greater for a larger array aperture than a smaller array aperture in applications where a large array aperture and a large beamwidth are required for high range and angle resolutions. Reference [9] has suggested the solution to the beam squint problem is to introduce time delay units (TDU) at the sub-array level, so as to match the delays caused by the signal arriving at the different elements at different times.

A TDU is inserted into the feed path of each element. The signals that pass through the TDU are delayed by time ΔT . By properly selecting the delay value feeding each element, the transmitted (or received) wave front is fixed in the desired direction at all frequencies [10]. However, an analog TDU is undesirable because it becomes large for significant time delays at low frequencies. It may be possible to add the necessary time delay in the signal processing step of a digital antenna.

D. ADAPTIVE BEAMFORMING

As discussed in the above sections, a phased array can not only beam steer to the desired direction for transmission (or reception) but also simultaneously suppress the radiation pattern in the directions of the interferers as shown in the Figure 5. This process is known as adaptive beamforming and nulling.

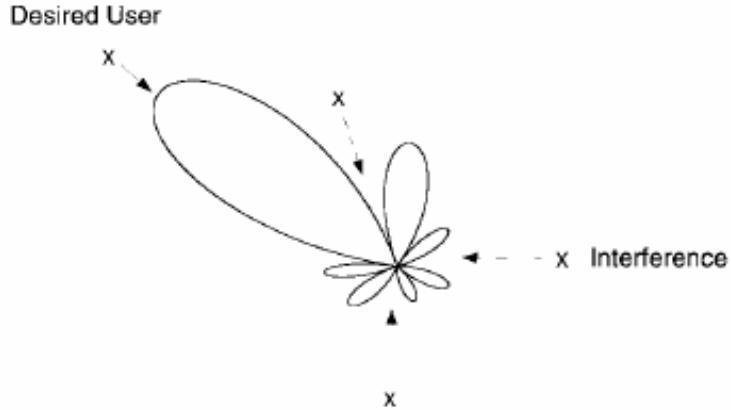


Figure 5. A Sample Array Pattern (From Ref. [11].).

Figure 5 shows that the mainbeam is steered to the desired user (or target) while at the same time, having nulls in the directions of the interferers. By doing so, the interference is minimized and the carrier-to-interference ratio for the desired user is maximized [11].

Reference [12] states that the maximum directivity of the linear array is proportional to the total length of the array (i.e., the length of the array can be increased by increasing the distance, usually described in terms of wavelength, between elements of the array). However there are some considerations for selecting the spacing between the antenna elements. If the antenna elements are spaced more than $\lambda/2$, grating lobes can appear, which cause undesirable effect on the radiation pattern of the array [12]. On the other hand, if the antenna elements are spaced less than $\lambda/4$, mutual coupling effects can occur that affect the radiation patterns and the impedance match. Thus the array performance will be degraded [12]. As a result, the separation dimensions between antenna elements typically ranges from $\lambda/4$ to $\lambda/2$.

E. DIRECTION FINDING

The basic principle of direction finding by an array antenna is described in the following paragraph using a two-element linear array model. The basic principle of a two-element linear array can then be extended to a larger number of elements. The details can be found in [13].

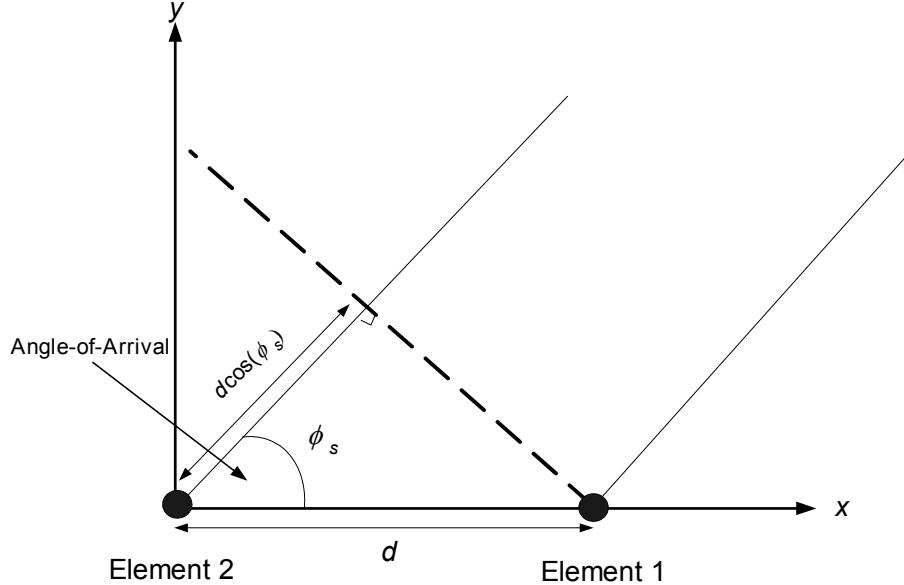


Figure 6. Two-Element Linear Array.

Figure 6 shows a two-element linear array positioned along the x -axis. Element two (2) receives the signal later than element one (1) as the wavefronts propagate toward the antenna with a propagation delay value $\tau_d = d \cos(\phi_s)/c$. Assume that the incoming narrowband signal has the following expression:

$$\begin{aligned} x(t) &= A(t) \cos(\omega_c t + \psi(t)) \\ &= A(t) \cos \psi(t) \cos(\omega_c t) - A(t) \sin \psi(t) \sin(\omega_c t) \\ &= x_1(t), \end{aligned} \tag{4}$$

where

$$\begin{aligned} A(t) &= \text{amplitude of incoming signal,} \\ \psi(t) &= \text{phase of incoming signal,} \\ \omega_c &= 2\pi f_c, \text{ and} \\ x_1(t) &= \text{signal received by element one.} \end{aligned}$$

Similarly the incoming signal received by element two has the following expression:

$$\begin{aligned} x_2(t) &= A(t + \tau_d) \cos(\omega_c(t + \tau_d) + \psi(t + \tau_d)) \\ &= A(t + \tau_d) \cos[\psi(t + \tau_d) + \omega_c \tau_d] \cos(\omega_c t) \\ &\quad - A(t + \tau_d) \sin[\psi(t + \tau_d) + \omega_c \tau_d] \sin(\omega_c t). \end{aligned} \quad (5)$$

The delay τ_d can be assumed a function of the carrier period, because the incoming signal is assumed to be narrowband. Thus Equation (5) is modified to the following expression:

$$x_2(t) = A(t) \cos[\psi(t) + \omega_c \tau_d] \cos(\omega_c t) - A(t) \sin[\psi(t) + \omega_c \tau_d] \sin(\omega_c t), \quad (6)$$

where

$$\begin{aligned} A(t) &\approx A(t + \tau_d) \text{ and} \\ \psi(t) &\approx \psi(t + \tau_d). \end{aligned}$$

The incoming signals are received by the receiver circuit for each of the elements as shown in Figure 7.

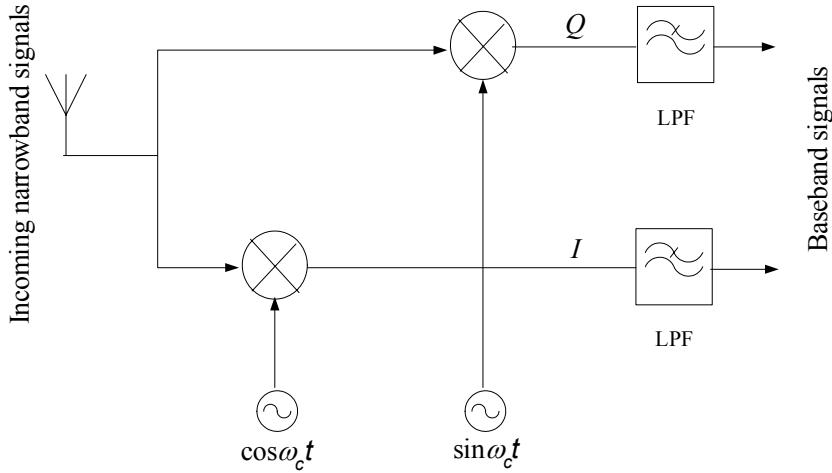


Figure 7. Receiver Circuit for an Element of the Array.

The narrowband incoming signals are demodulated to their respective baseband signals by the receiver circuit as shown in the following expressions for each of the elements. The I and Q are referred to as the in-phase and quadrature terms, respectively.

For element one, we can write:

$$\begin{aligned} I_1(t) &= A(t) \cos(\psi(t)) \\ Q_1(t) &= A(t) \sin(\psi(t)) \end{aligned} \tag{7}$$

while for element two:

$$\begin{aligned} I_2(t) &= A(t) \cos(\psi(t)) \cos(kd \cos(\phi_s)) - A(t) \sin(\psi(t)) \sin(kd \cos(\phi_s)) \\ Q_2(t) &= A(t) \cos(\psi(t)) \sin(kd \cos(\phi_s)) - A(t) \sin(\psi(t)) \cos(kd \cos(\phi_s)) \end{aligned} \tag{8}$$

where $kd \cos(\phi_s) = \omega_c \tau_c$.

The phase of the Local Oscillator (LO) in the receiver is adjusted such that $I_1(t)$ is maximized and $Q_1(t)$ is zero. The receiver is synchronized at the reference element with the incoming signal. Thus, in such a case $\psi(t) = 0$, which simplifies the expressions to

$$\begin{aligned}
I_1(t) &= A(t) \\
Q_1(t) &= 0 \\
I_2(t) &= A(t) \cos(kd \cos(\phi_s)) \\
Q_2(t) &= A(t) \sin(kd \cos(\phi_s)).
\end{aligned} \tag{9}$$

As a result, the ratio of the two in-phase signals can be used to determine the angle of arrival

$$\begin{aligned}
\frac{I_2(t)}{I_1(t)} &= \frac{A(t) \cos(kd \cos(\phi_s))}{A(t)} \\
&= \cos[kd \cos(\phi_s)]
\end{aligned} \tag{10}$$

which gives

$$\phi_s = \cos^{-1}\left(\frac{1}{kd} \cos^{-1}\left(\frac{I_2(t)}{I_1(t)}\right)\right). \tag{11}$$

Equation (11) assumes that the carrier frequency is known a-priori and is noise free. Also, that the $I_{1,2}(t)$ and $Q_{1,2}(t)$ values can be measured from the receiver. The drawback of a linear array is that the ϕ_s has the front-back ambiguity (i.e., two possible solutions).

F. *I AND Q AND DEMODULATION SCHEME*

Equation (7) can be used to demonstrate the *I* and *Q* demodulation scheme. In-phase and quadrature detection is able to recover a baseband signal without losing any information, by using Equation (7) [14]. The amplitude and phase can be recovered by

$$A = \sqrt{I^2 + Q^2} \tag{12}$$

and

$$\psi = \tan^{-1}\left(\frac{Q}{I}\right). \tag{13}$$

G. DIGITAL ANTENNA ARCHITECTURE

The basic architecture of a digital phased array transmit antenna is shown in Figure 8.

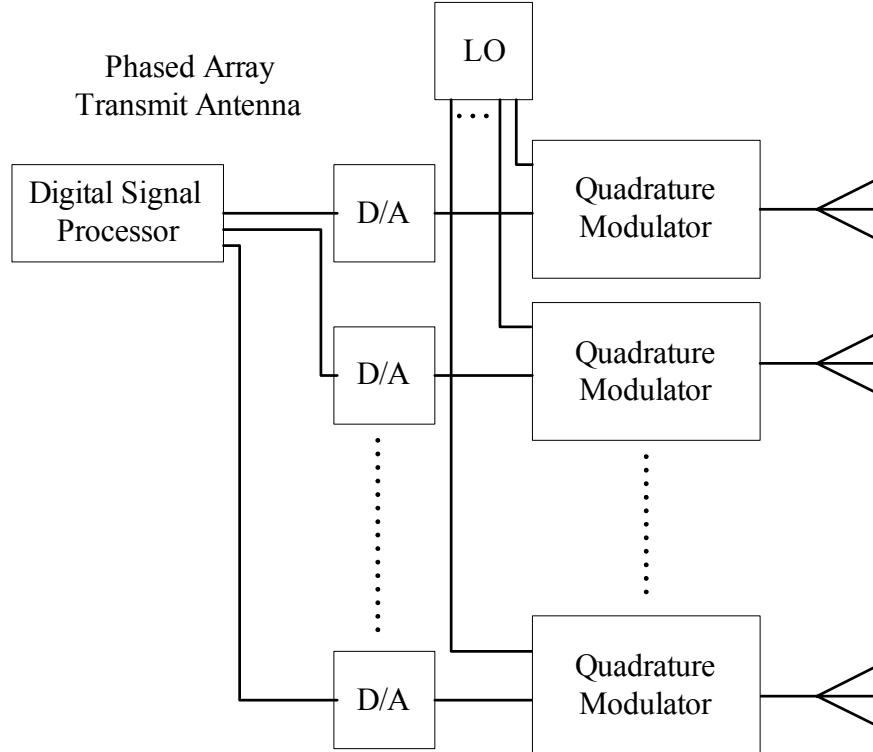


Figure 8. Architecture of a Digital Phased Array Transmit Antenna.

The transmit antenna is comprised of (1) a Digital Signal Processor (DSP), (2) multiple Digital-to-Analog Converters (DACs), (3) multiple modulator boards, (4) a Local Oscillator (LO) generator, and (5) multiple antenna elements.

The DSP is the brain of the digital antenna; it has the algorithms to compute the necessary complex coefficients required for beamforming. The data from the DSP is then converted from digital format to analog format before sending to the multiple modulators. The modulator, which performs as a phase shifter and amplitude controller in the digital antenna, modulates the analog data from the DAC with a carrier signal. The carrier signals for the multiple modulators are phase coherent and supplied by a LO generator. The phased shifted signals are then fed to the multiple antenna elements for transmission.

The digital phased array receive antenna performs the reverse function of the transmit antenna, and the basic architecture is shown Figure 9. The LO generator supplies the LO carrier signal to the demodulators in order for the demodulators to down convert the received signals to baseband signals. The baseband signals are then converted from analog format to digital format by the Analog-to-Digital Converters (ADCs) before sending them to the DSP for processing.

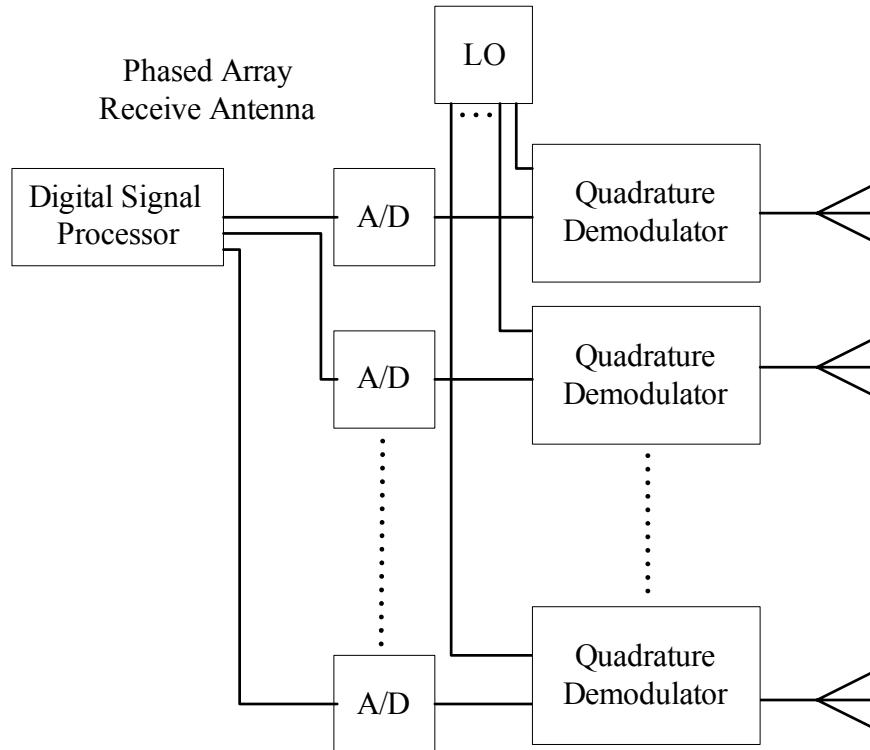


Figure 9. Architecture of a Digital Phased Array Receive Antenna.

H. SUMMARY

This chapter has presented the basic concept of arrays and also discussed some of the capabilities that the arrays have. This research focuses on the application of commercial modulators and demodulators to perform as phase shifters as described Section G. In addition, Equation (2) will be used in Chapter III, Section 4 to generate the radiation patterns of a linear array using MathCAD.

In the next chapter, both the commercial modulator and demodulator boards are experimentally configured, and their measured amplitudes and phases are obtained for analysis.

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III. DESIGN SELECTION AND MEASUREMENT ANALYSIS

This chapter provides the analysis of the measured scattering parameters (S_{21}) for the phased array transmit antenna in terms of amplitude and phase. In addition, these measured data are used to verify whether the modulator board has wideband performance by simulating radiation patterns of a linear phased array. The demodulator board for the phased array receiver is characterized and the results are presented.

A. THREE-DIMENSIONAL 2.4-GHZ PHASED ARRAY TRANSMIT ANTENNA

A 3-D 2.4-GHz phased array transmit antenna was designed and built by Lance C. Esswein [3]. The intent of his research was to assess whether the GA could be used as a design and beamforming tool for transmitting phased arrays, and whether commercially available RF microchips could be used to build the transmitting phased array antenna. The measurement results obtained from the constructed phased array antenna fully met the stated objectives. The details of his thesis can be found in [3].

In this thesis, the bandwidth characteristics (i.e., phase versus frequency response, and amplitude versus frequency response) of the Analog Devices AD8346EVAL Quadrature Modulator board are investigated.

1. AD8346EVAL Quadrature Modulator

The AD8346EVAL board is a silicon RF Integrated Circuit (RFIC) I/Q modulator that is normally used as the transmit modulator in digital systems such as Code Division Multiple Access (CDMA) and Global Satellite Mobile (GSM) transceivers. However, in [3], this modulator board was configured as a phase shifter for use in a digital 3-D phased array antenna.

The AD8346EVAL board has an operating frequency range from 0.8 GHz to 2.5 GHz [15] and, as such, this feature makes the modulator a potential component in wideband antennas for many sensor and communication applications.

2. Operation of the AD8346EVAL Board as a Phase Shifter

The following paragraph describes the controlling information necessary to operate the AD8346EVAL board as a phase shifter.

There are four input pins used for phase control namely (1) in-phase negative (IN), (2) in-phase positive (IP), (3) quadrature negative (QN), and (4) quadrature positive (QP). The phase shift quadrant determines which two pins receive positive control voltage signals while the other two pins are set to zero according to [3]. Figure 10 indicates the control pin combinations used in each of the quadrants while Table 1 shows the amount of control voltage needed for each pin in order to set the appropriate phase.

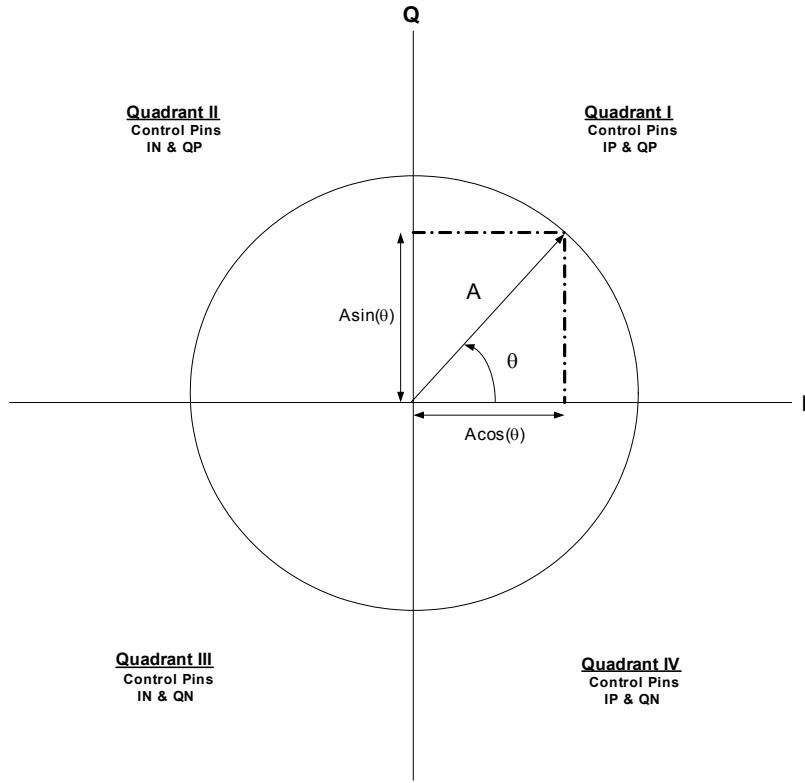


Figure 10. Complex (Phasor) Plane (From Ref. [3].).

Signal Phase Shift	IN	IP	QP	QN
000-090	0	$\cos(\theta)$	$\sin(\theta)$	0
090-180	$\cos(\theta)$	0	$\sin(\theta)$	0
180-270	$\cos(\theta)$	0	0	$\sin(\theta)$
270-360	0	$\cos(\theta)$	0	$\sin(\theta)$

Table 1. Control Signal Value for Phase Shift (From Ref. [3].).

3. Magnitude and Phase Measurements for the AD8346EVAL Board

The AD8346EVAL board is configured as shown in Figure 11 to take the magnitude and phase measurements using a HP8510 Vector Network Analyzer (VNA).

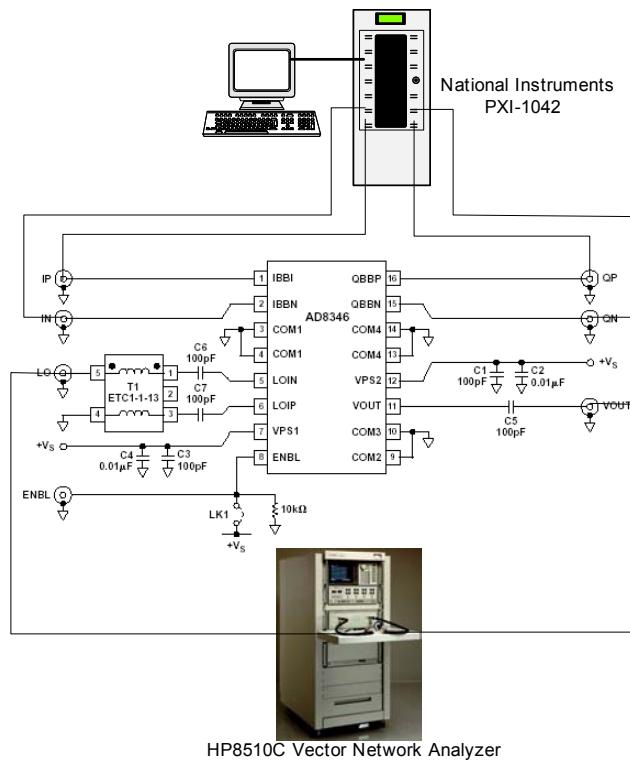


Figure 11. AD8346EVAL Bench Test Configuration.

The control voltages for the four input pins (i.e., IN, IP, QN and QP) of the AD8346EVAL board are supplied by the National Instruments PXI-1042, while a control program written in LabVIEW version 6.1 is used to control the amount of phase shift to the AD8346EVAL board. Figure 12 displays the AD8346EVAL cable connections while

Figure 13 shows a view of the control program Graphic User Interface (GUI) written in LabVIEW. In addition, Figure 14 shows the radar equipment cart containing the DSP (NI PXI-1042), the LO generator, and the multiple AD8346EVAL modulator boards.

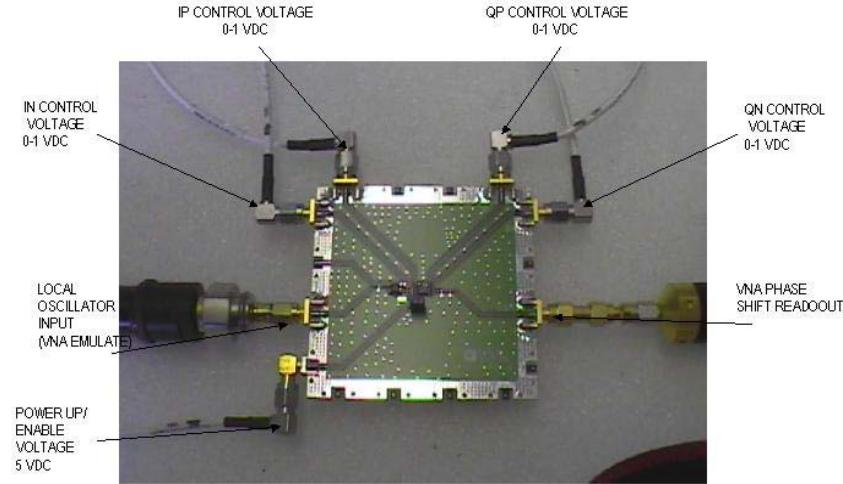


Figure 12. AD8346EVAL Cable and Signal Connections (From Ref. [3].).

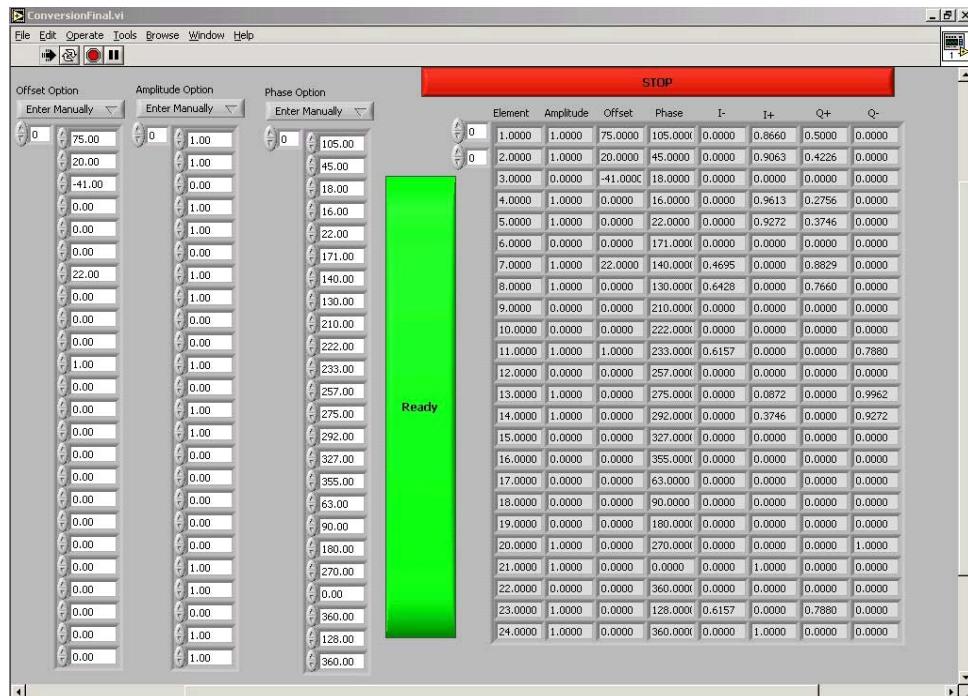


Figure 13. The Control Program GUI.

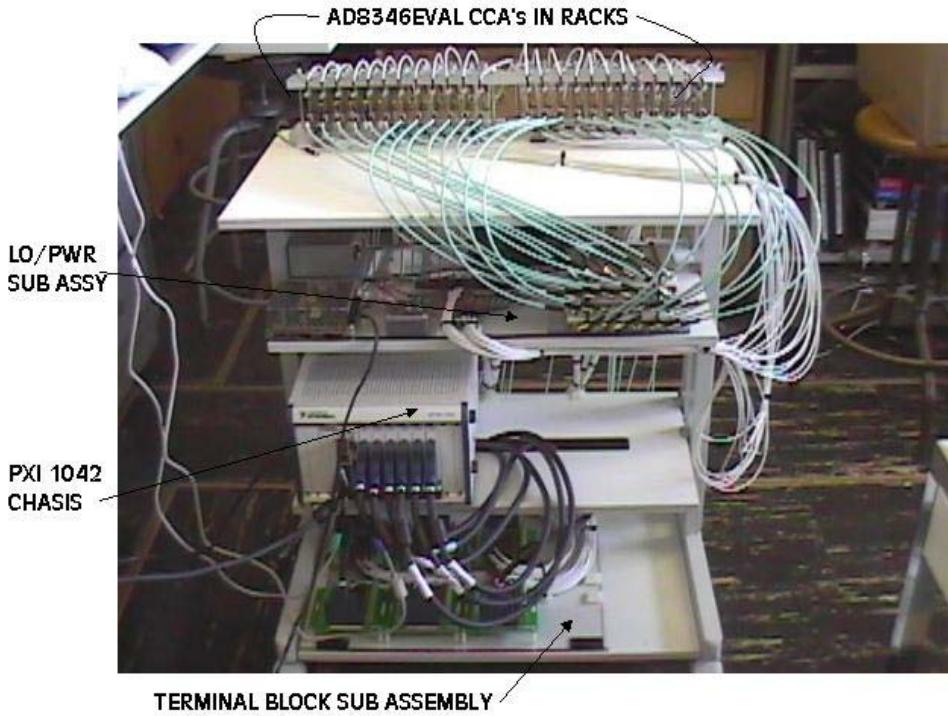


Figure 14. Radar Equipment Cart (From Ref. [3].).

One of the twenty-four inputs in the control program is used to set the voltage supply to the four input pins of the AD8346EVAL board for each element. The VNA supplies the local oscillator source signal and takes the measurements of the insertion phase shift and loss in the signal, as the control voltages are adjusted. The VNA is programmed to have a frequency sweep from 0.8 GHz to 2.5 GHz with an input power of 10 dBm. The S_{21} parameter for the AD8346EVAL board is then measured and saved. (The magnitude of S_{21} in dB is the insertion loss; the phase of S_{21} is the insertion phase.)

Figures 15 and 16 show the S_{21} parameter magnitude and phase measurements when the AD8346EVAL board is set to zero-phase by the control program.

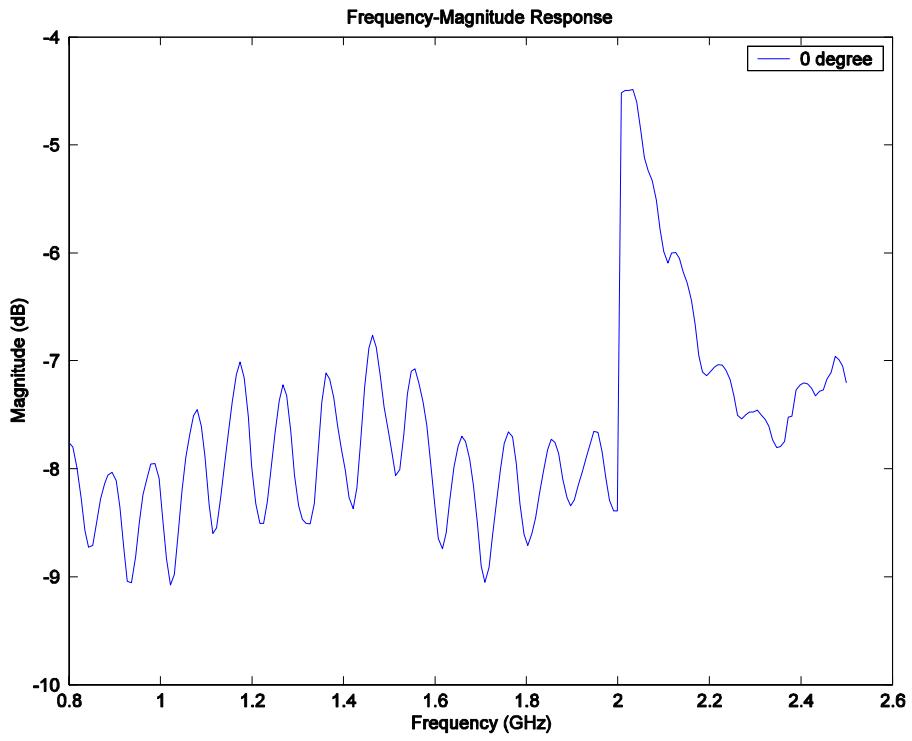


Figure 15. Frequency-Magnitude Response for Zero Phase Setting.

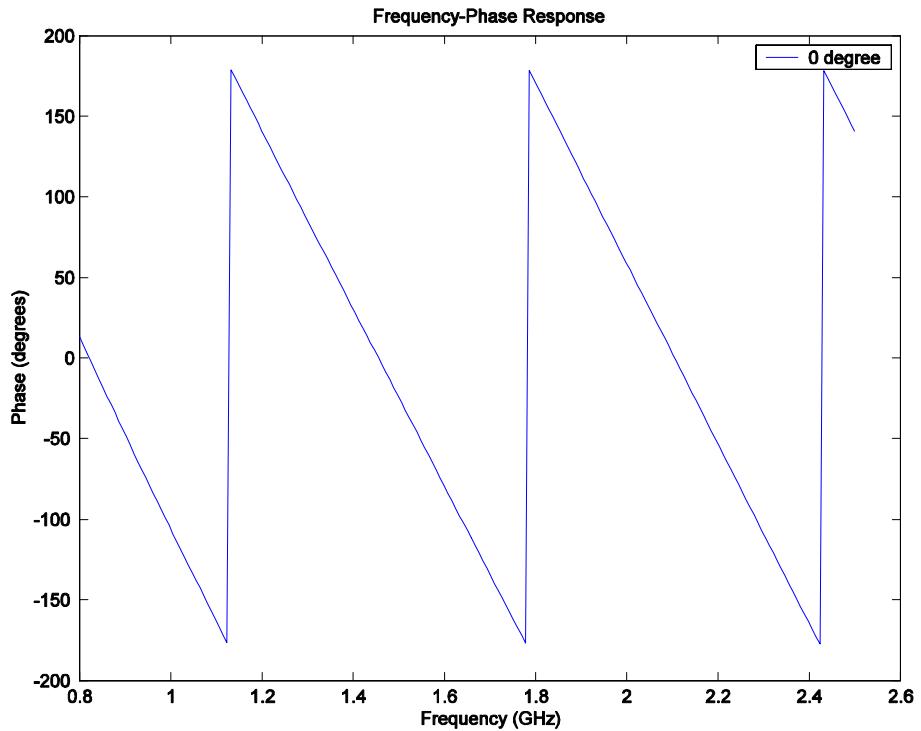


Figure 16. Frequency-Phase Response for Zero Phase setting.

Figure 15 shows that the AD8346EVAL board has an insertion loss that ranges from -6.5 dB to -9 dB in the frequency band from 0.8 GHz to 2 GHz. The magnitude increases to -4.5 dB from 2.0 GHz to 2.2 GHz. Figure 16 shows that the board has a linear phase slope from $-\pi$ to $+\pi$ radians. The calibration for the VNA was a direct connection, thus there are several cycles of phase as shown in the Figure 16. Note that the phase slope is negative, which is the opposite of what occurs for a passive device. A transmission line of length d has a phase given by $2\pi fd/c$, which has a positive phase slope.

The AD8346EVAL board is set for a 360° sweep in 1° increment and the S_{21} parameter measured at each 1° increment is saved. These measured data files are used for simulation of the array antennas in Chapter III, Section 4. The results of the measurements are plotted at 5° increments, in order to verify that the AD8346EVAL board phase slopes and magnitudes track as a function of phase state. Figures 17 to 20 show the results for phase shifts from 0° to 75° , 90° to 165° , 180° to 255° , and 270° to 345° , respectively. The modulo 360° phase measurements taken from the VNA are converted into linear continuous straight lines for ease of comparison. As expected, the phase shifts are observed to occur in sequence when the phase shift is changed sequentially from one value to another. For example, from Figure 17, the frequency-phase response at 30° phase shift is observed to be on top of the frequency-phase response at 15° phase shift. From the figures, it is clear that the frequency versus phase response of the AD8346EVAL board has a linear negative phase slope that tracks the phase shift settings.

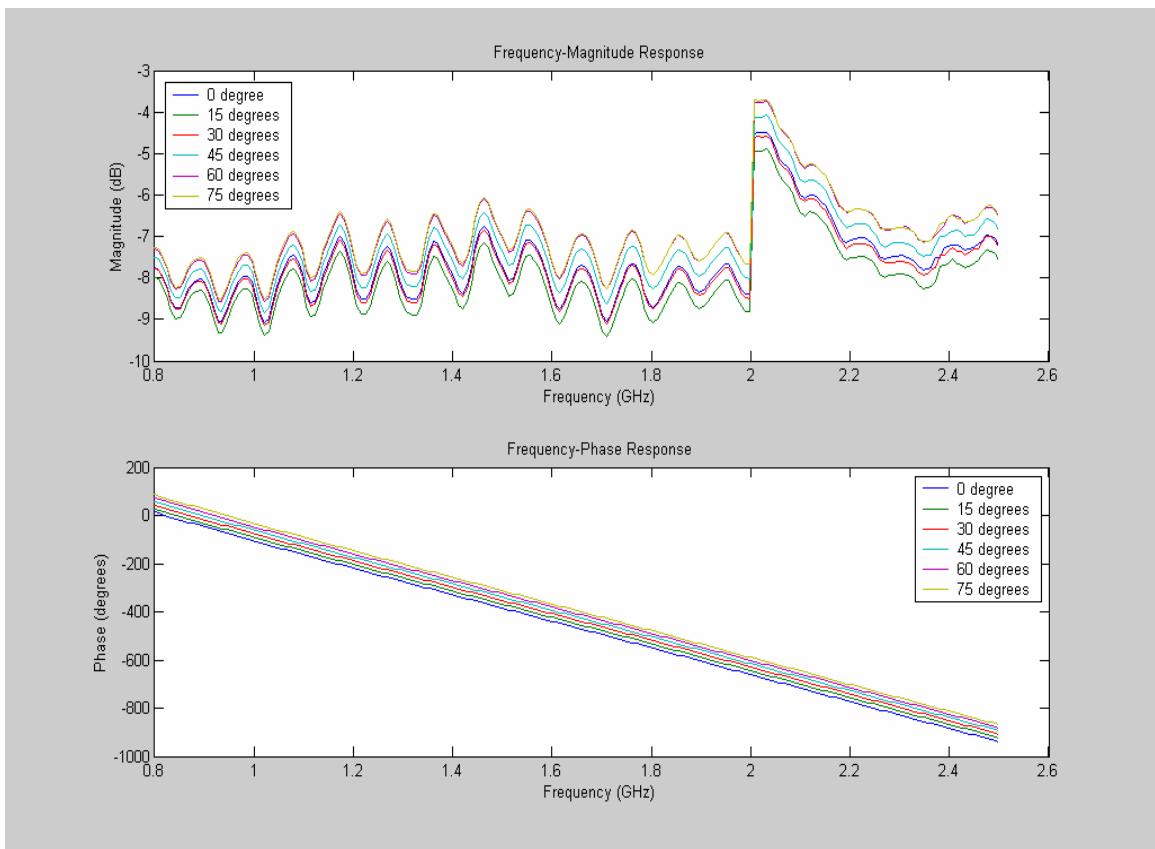


Figure 17. Magnitude and Phase Versus Frequency Response for Phase Shift From 0° to 75° at 5° Increments.

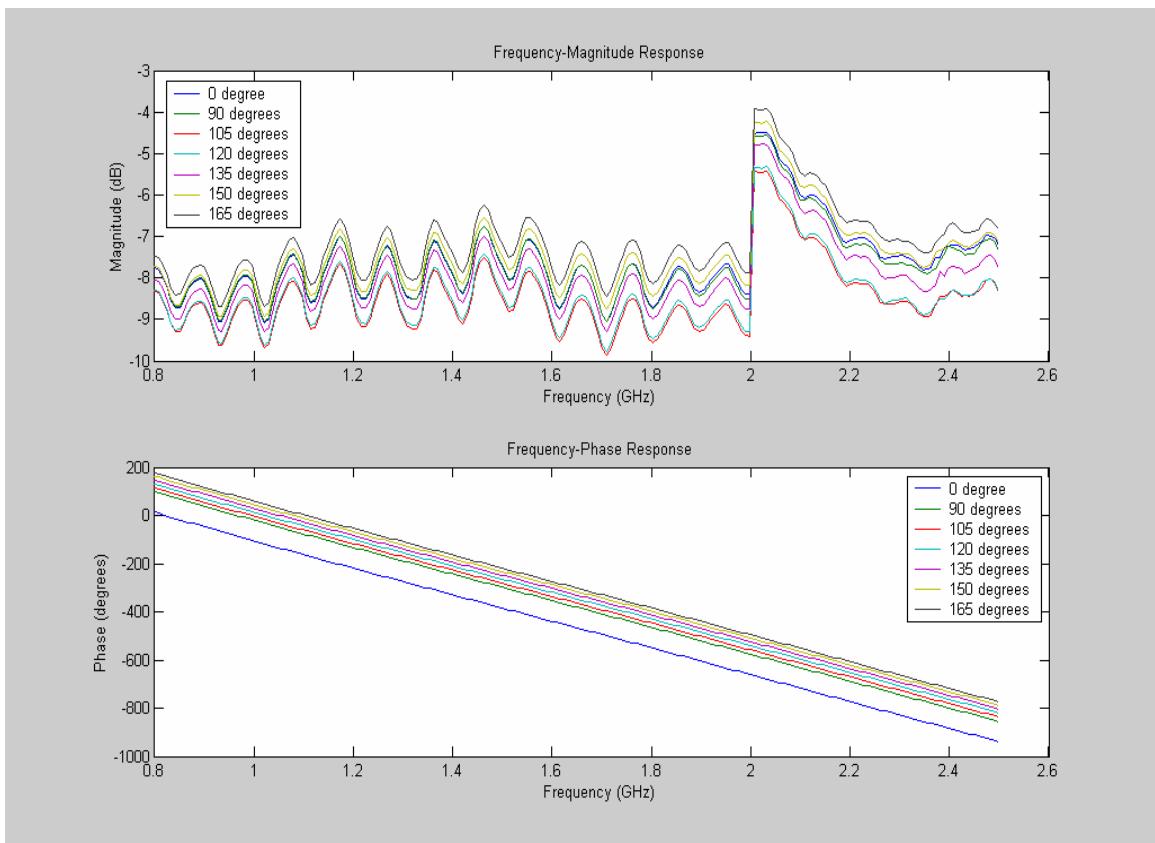


Figure 18. Magnitude and Phase Versus Frequency Response for Phase Shift From 90° to 165° at 5° Increments. (The 0° Phase Shift is Plotted for Reference.)

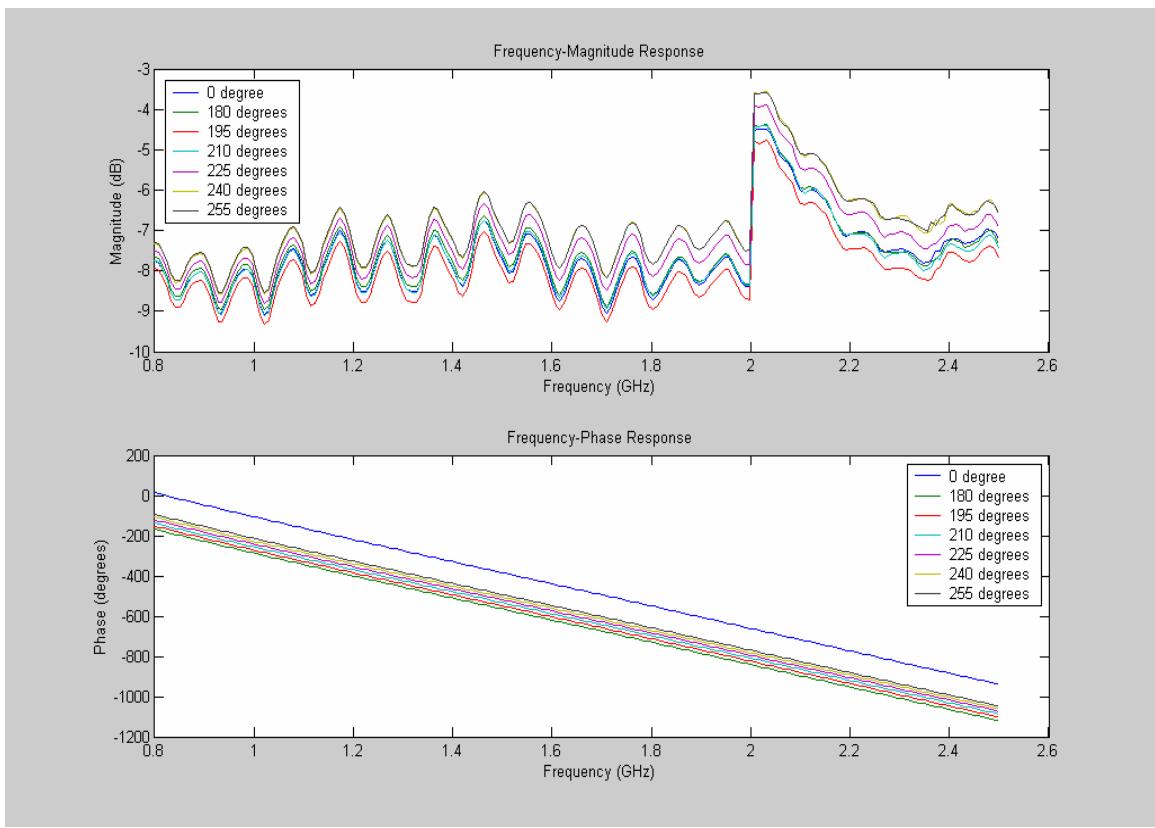


Figure 19. Magnitude and Phase Versus Frequency Response for Phase Shift From 180° to 255° at 5° Increments. (The 0° Phase Shift is Plotted for Reference.)

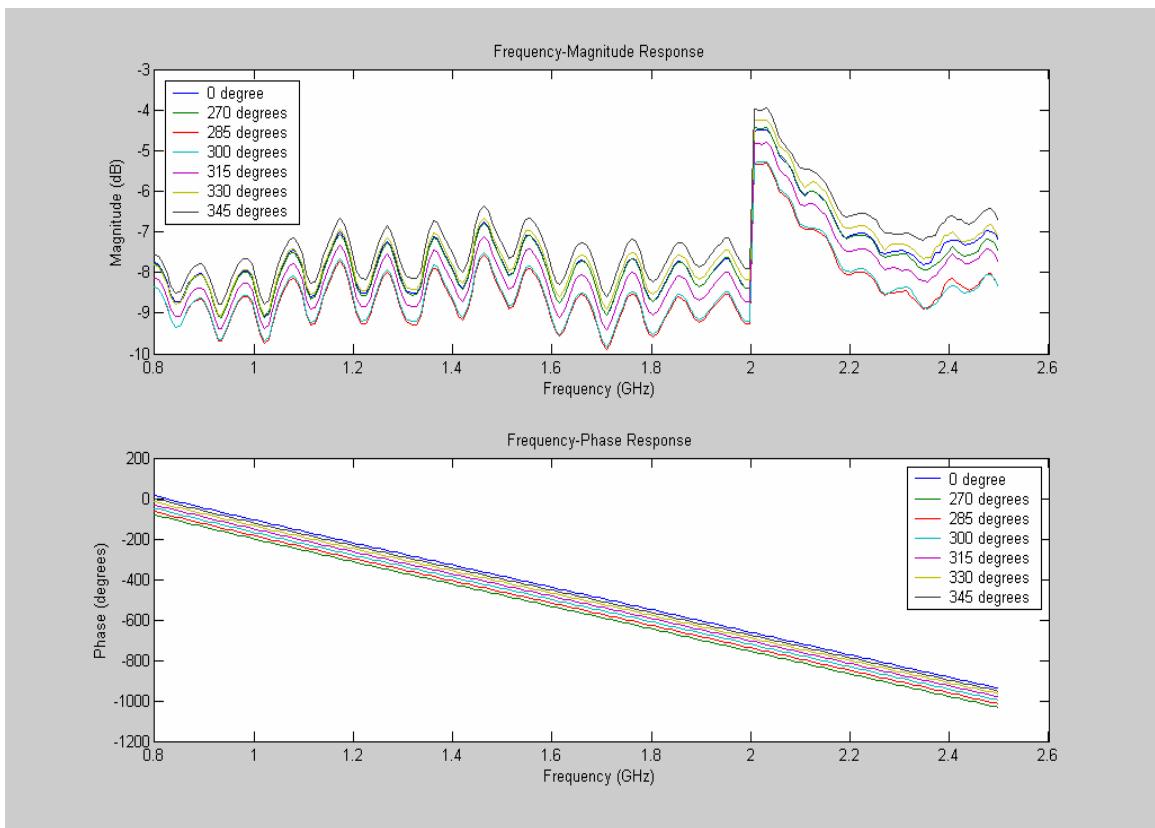


Figure 20. Magnitude and Phase Versus Frequency Response for Phase Shift From 270° to 345° at 5° Increments. (The 0° Phase Shift is Plotted for Reference.)

4. Implementation of a Linear Array

A simulation program was written to investigate whether the AD8346EVAL board is capable of providing wideband pattern characteristics, since the board has operating frequency from 0.8 GHz to 2.5 GHz. The simulation program was implemented in MathCAD and used the measured S_{21} parameters collected previously. A thirty-element linear array was considered and the radiation patterns at various operating frequencies were plotted for comparison.

a. Parameter Specifications

The simulation is based on the linear array shown in Figure 21. The physical spacing d between elements has to be fixed at a value that is based on a reference operating frequency. The reference frequency of 1.7 GHz is chosen in the middle of the operating frequency range of the AD8346EVAL board. Thus, the set of phase settings is computed at 1.7 GHz and held fixed for all frequencies. The operating parameters for the simulation program are as follows:

Number of elements, $N = 30$,

Reference frequency, $f_{ref} = 1.7$ GHz,

Reference wavelength, $\lambda_{ref} = c/f_{ref} = 0.176$ m,

Array spacing, $d = \lambda_{ref}/2 = 0.088$ m,

The reference frequency wave number, $k_{ref} = 2\pi/\lambda_{ref} = 35.605$, and

The angle of transmission (scan angle), $\theta_s = 10^\circ$.

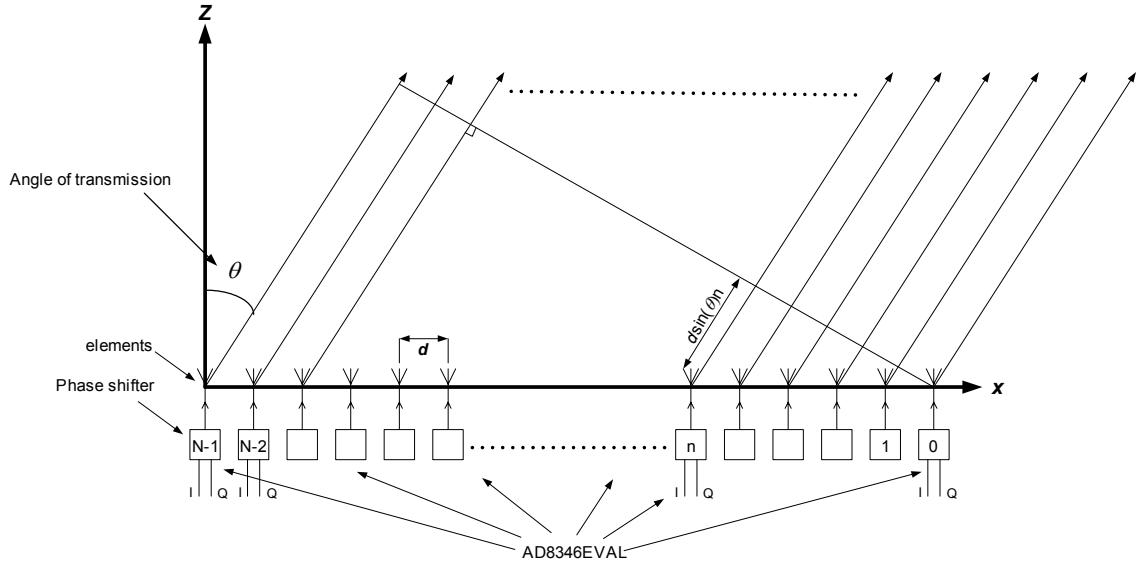


Figure 21. Thirty-Element Linear Array.

b. Phase Reference and Phase Shifts

Element 0 is chosen as the phase reference as shown in Figure 21. Based on this reference element, the phase shift setting at each element is given by Equation (14). The phase shift increment at element $n + 1$ with respect to element n is

$$\begin{aligned}\Delta\Phi &= k_{ref}d \sin \theta_s \\ &= 35.605 \times 0.088 \times \sin(10^\circ) \\ &= 31^\circ.\end{aligned}\quad (14)$$

The phase shift associated with each element is shown in the Table 2.

Element	Phase Shift (°)	Element	Phase Shift (°)	Element	Phase Shift (°)
0	0	10	310	20	260
1	31	11	341	21	291
2	62	12	12	22	322
3	93	13	43	23	353
4	124	14	74	24	24
5	155	15	105	25	55
6	186	16	136	26	86
7	217	17	167	27	117
8	248	18	198	28	148
9	279	19	229	29	179

Table 2. Phase Shift Associated With Each Element.

The simulation program reads the corresponding measured data file based on the phase shifts listed in Table 2 for each element. For example, the 93° data file is read in for element 3. Figure 22 shows the Frequency-Phase and Frequency-Magnitude Responses for the 93° phase state. At any operating frequency, the corresponding phase and magnitude are read from the plot. For example, at $f_{oper} = 1.2$ GHz, the phase is -130° and the magnitude is -7.1 dB. Hence, element 3 is set with phase -130° and the amplitude at -7.1 dB for $f_{oper} = 1.2$ GHz. Therefore, the actual measured data are used in calculating the array factor at a given frequency.

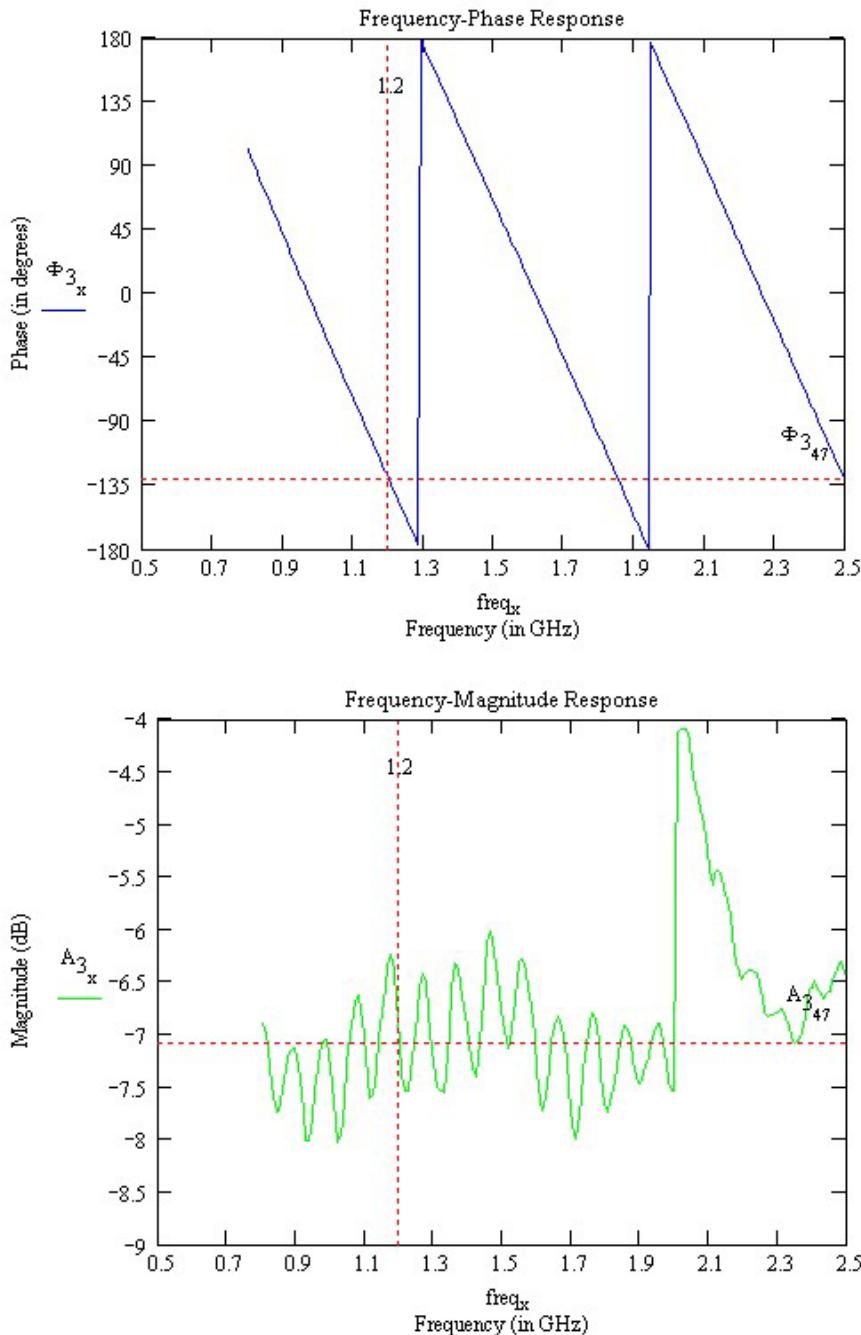


Figure 22. Frequency-Phase Response for a 93° Phase Shift Setting.

c. *Radiation Patterns*

With the calculated phase shift at each element, the program reads in the corresponding measured phase and amplitude data files for the AD8346EVAL board. Using Equation (1), the radiation pattern of a linear array at various operating frequencies

can be plotted. The radiation pattern for operating frequency $f_{oper} = 1.7$ GHz is shown in Figure 23.

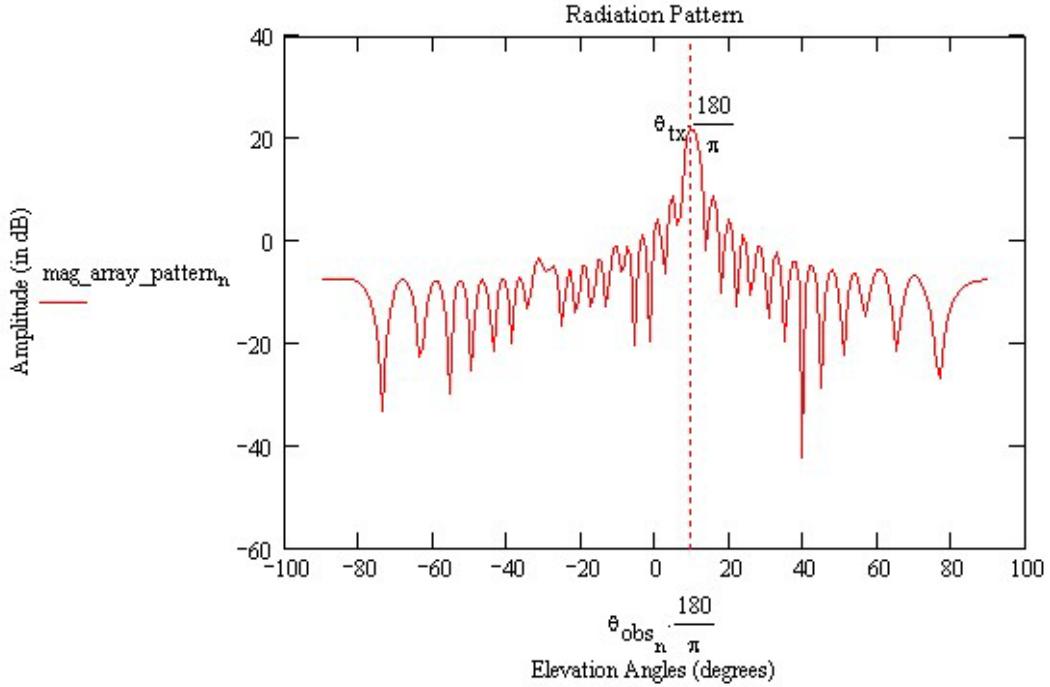


Figure 23. Radiation Pattern at $f_{oper} = 1.7$ GHz.

Figure 23 shows that the main beam of the linear array is scanned to the direction of transmission, i.e., $\theta_s = 10^\circ$. However when the operating frequency is changed to 0.8 GHz, the mainbeam is squinted about $+11^\circ$ off of the direction of transmission, as shown in Figure 24. When the operating frequency is changed to 2.5 GHz, the mainbeam is squinted about -5° off of the direction of transmission as shown in Figure 25. Furthermore, because the phase slope of the modulator board is negative, the beam scans toward endfire as the frequency is decreased. This is the opposite of what would occur if a passive beamforming network is used.

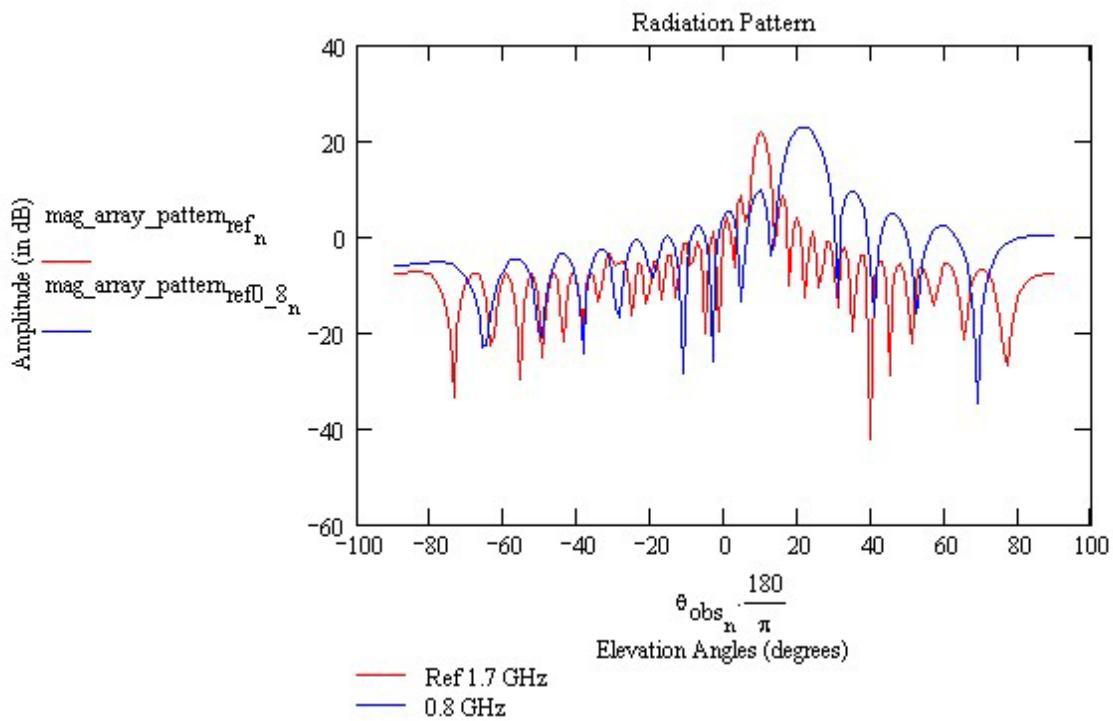


Figure 24. Radiation Pattern at $f_{\text{oper}} = 0.8$ GHz Compared to $f_{\text{ref}} = 1.7$ GHz.

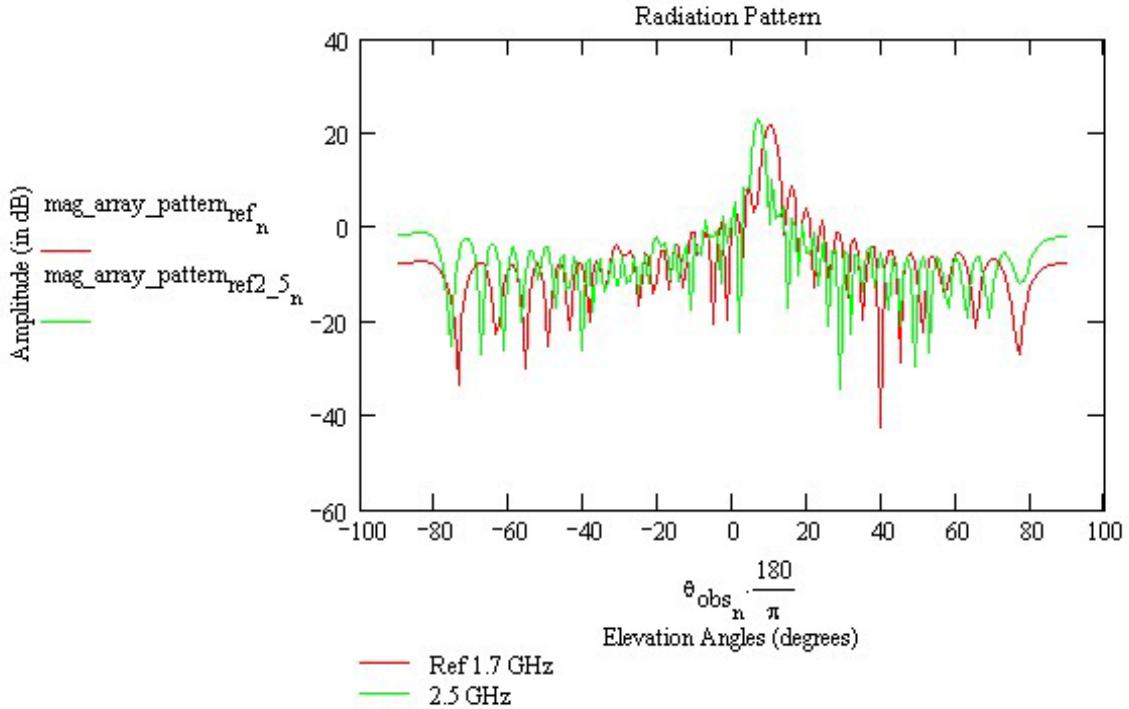


Figure 25. Radiation Pattern at $f_{oper} = 2.5$ GHz Compared to $f_{ref} = 1.7$ GHz.

The half-power beamwidth of the radiation pattern changes according to [16]

$$\theta_B \approx \frac{0.866c}{Nd_{oper} \cos(\theta_s)} \cdot \frac{180}{\pi}. \quad (15)$$

Thus, the beamwidth of the radiation patterns are broader (i.e., see Figure 24) and narrower (i.e., see Figure 25) when the operating frequencies are 0.8 GHz and 2.5 GHz, respectively. As evident from radiation patterns, the AD8346EVAL board does not have a wide instantaneous bandwidth characteristic. If the transmitted waveform has frequency components between 0.8 and 2.5 GHz, then the low frequencies are radiated in a different direction than the high frequencies.

B. PHASED ARRAY RECEIVER

The next phase of the project was to develop a complementary receive array. This would allow the GA to be verified for the entire beam forming process from transmitting to reception. Three approaches were considered in developing a receive digital phased array. These options are described in the following paragraphs.

1. Receiver Component Design Considerations

a. *Option One*

In option one, twenty-four RF Signal Analyzers from National Instruments (NI) are considered, with each an RF Signal Analyzer feeding to one antenna element of the twenty-four elements used in the array receiver, as shown in Figure 26.

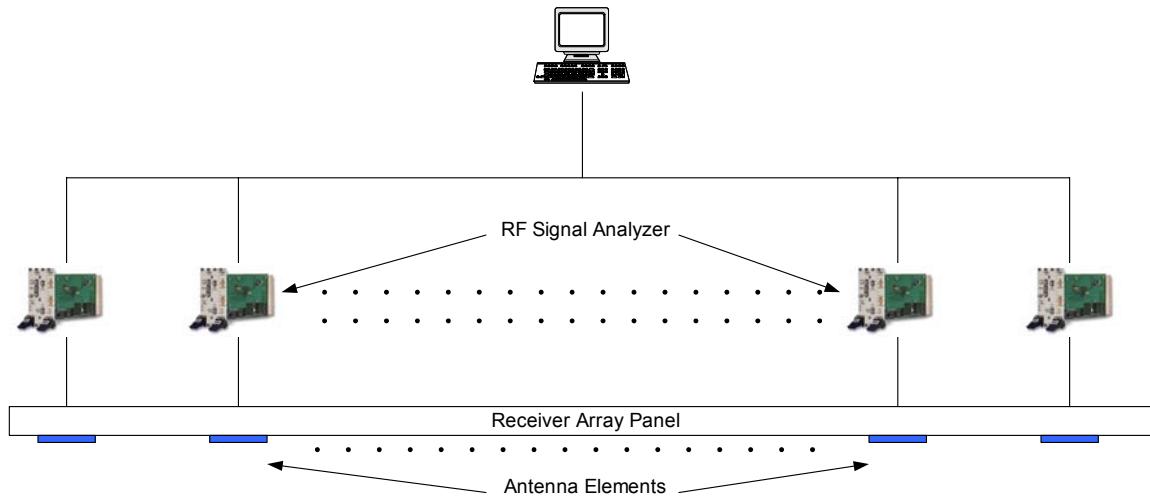


Figure 26. Option One for Array Receiver.

The RF Signal Analyzer is readily available and the software program to be written in LabVIEW program will be compatible with the 3-D 2.4-GHz Phased Array Transmit Antenna. However this is a very costly solution because the RF signal analyzer is a high cost item (\$11,000 each).

b. Option Two

In option two, two of the RF Signal Analyzers proposed in option one are used, each controlling three National Instruments ADCs switches as shown in Figure 27.

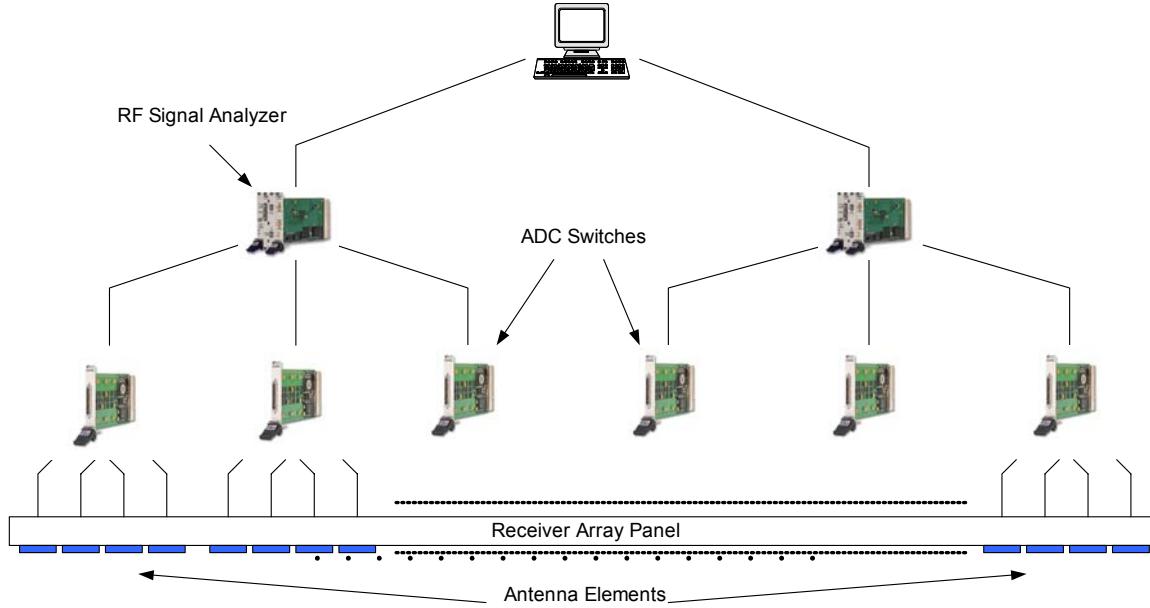


Figure 27. Option Two for Array Antenna.

There are six NI ADCs used in the architecture of the array receiver. Each ADC controls four antenna elements of the twenty-four antenna elements in the array. Although this option is cheaper than option one, it is still too costly due to the use of the RF Signal Analyzers.

c. Option Three

In option three, COTS I/Q demodulator boards performing as phase shifters is proposed. As shown in Figure 28, each of the demodulator boards feeds one antenna element. The output of the demodulator card is connected to a COTS DAC, which in turn is connected to a NI low-cost Input/Output (I/O) board. These I/O boards will be controlled by one RF Signal Analyzer.

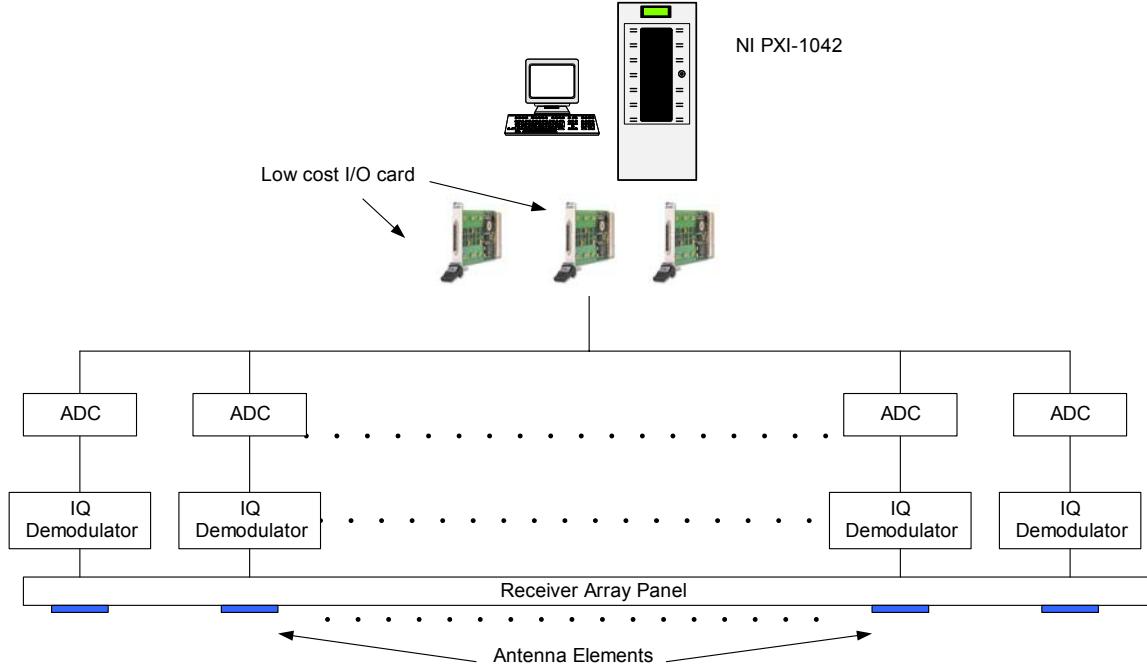


Figure 28. Option Three for Array Antenna.

The cost of the demodulator is one hundred dollars while the DAC cost about two hundred dollars. Thus option three is considered the cheapest among the three options. However, these demodulators have to be verified as to whether they can perform as phase shifters in the receive array antenna.

The AD AD8347EVAL Quadrature Demodulator board was selected as a possible candidate for the phase shifter. The AD8347EVAL board is the receive version of the AD8346EVAL board used in the 3-D 2.4 GHz Phased Array Transmit Antenna.

2. AD8347EVAL Quadrature Demodulator

The AD8347EVAL board is a silicon RFIC broadband direct quadrature demodulator with RF and baseband Automatic Gain Control (AGC) amplifiers. It is fully complementary with the AD8346EVAL modulator board for use in many communications receivers, performing quadrature demodulation directly to baseband frequencies. Figure 29 shows the electrical schematic of the AD8347EVAL microcircuit.

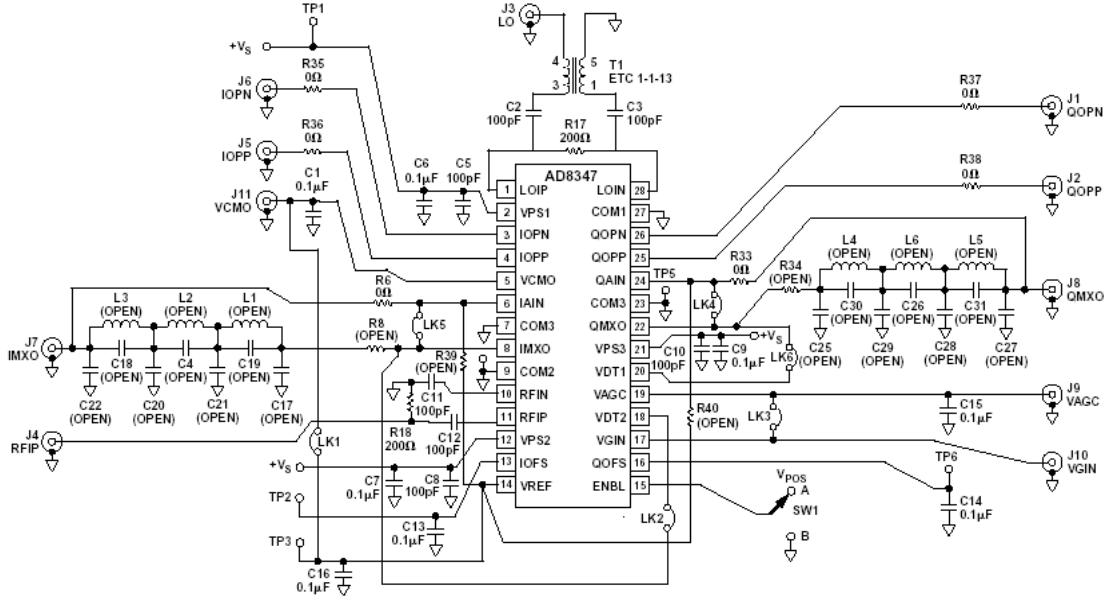


Figure 29. AD8347EVAL Circuit Diagram (From Ref. [17].).

The AD8347EVAL board RF input frequency ranges from 0.8 GHz to 2.7 GHz and the baseband outputs can be connected directly to an ADC. The RF signal is directly down converted to the baseband *I* and *Q* components using a LO signal at the same frequency as the RF signal. Direct conversion RF receiver architecture promises superior performances in power, size and cost over that of a superheterodyne based receiver [18].

There are thirteen signals lines in the form of SMA connectors in the AD8347EVAL board of which six signals are used in the bench test configuration described in Section 3. There are the Radio Frequency Input Positive (RFIP), Local Oscillation (LO), In-phase Output Pin Negative (IOPN), In-phase Output Pin Positive (IOPP), Quadrature Output Pin Negative (QOPN) and Quadrature Output Pin Positive (QOPP).

The RFIP and LO signal lines are configured for single-ended connections while the baseband outputs (IOPN, IOPP, QOPN, and QOPP) are configured for differential connections. The rest of the SMA connectors are mainly used for monitoring of the AD8347EVAL board.

3. AD8347EVAL Quadrature Demodulator Bench Test Configuration

A bench test was setup to investigate the operating characteristics of the AD8347EVAL board. The bench test configuration is shown in Figure 30.

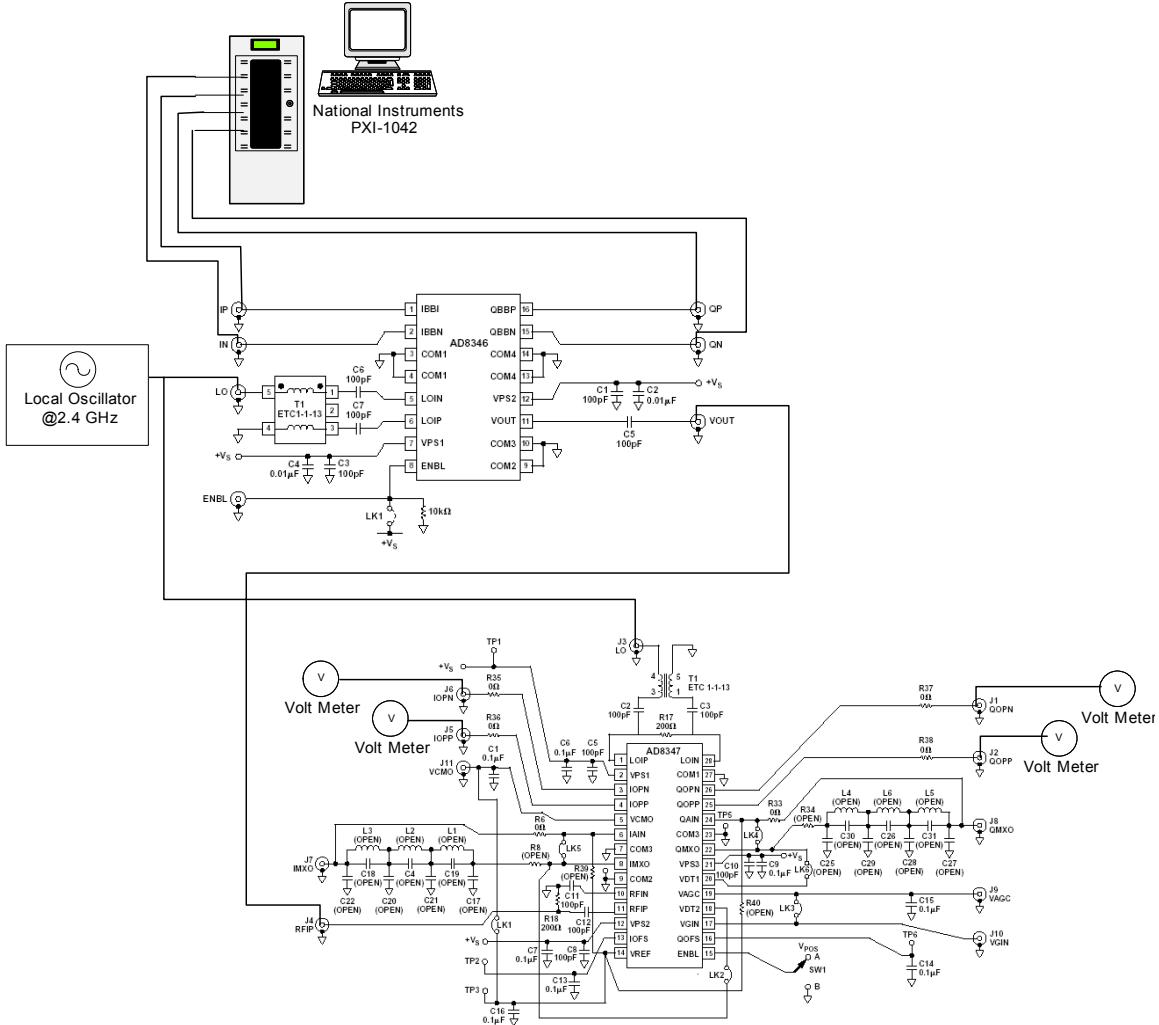


Figure 30. AD8347EVAL Bench Test Configuration.

A 5 VDC is supplied to the AD8347EVAL board by an external dc power supply. The positive and negative voltages of the dc power supply are connected to the pins “VS” and “GND” as shown in Figure 31. The location of the power ON/OFF switch is shown in Figure 31 as well. One of the AD8346EVAL transmit boards supplied the phase shifted RF signal to the RFIP of the AD8347EVAL board. The same LO generator operating at 2.4 GHz, used in the transmit antenna, is also used to supply the LO signal to the AD8347EVAL board. Hence the received RF signal is perfectly synchronized with

the LO signal at the AD8347EVAL board. The connections for the RF and LO signals can be seen in Figure 31. According to Reference 17, the operating range of the LO signal is between -10 and 0 dBm, whereas the signal supply by the LO generator is 12 dBm. Thus a RF attenuator (19 dB) has to be connected to the LO of the AD8347EVAL board before connecting the signal from the LO generator.

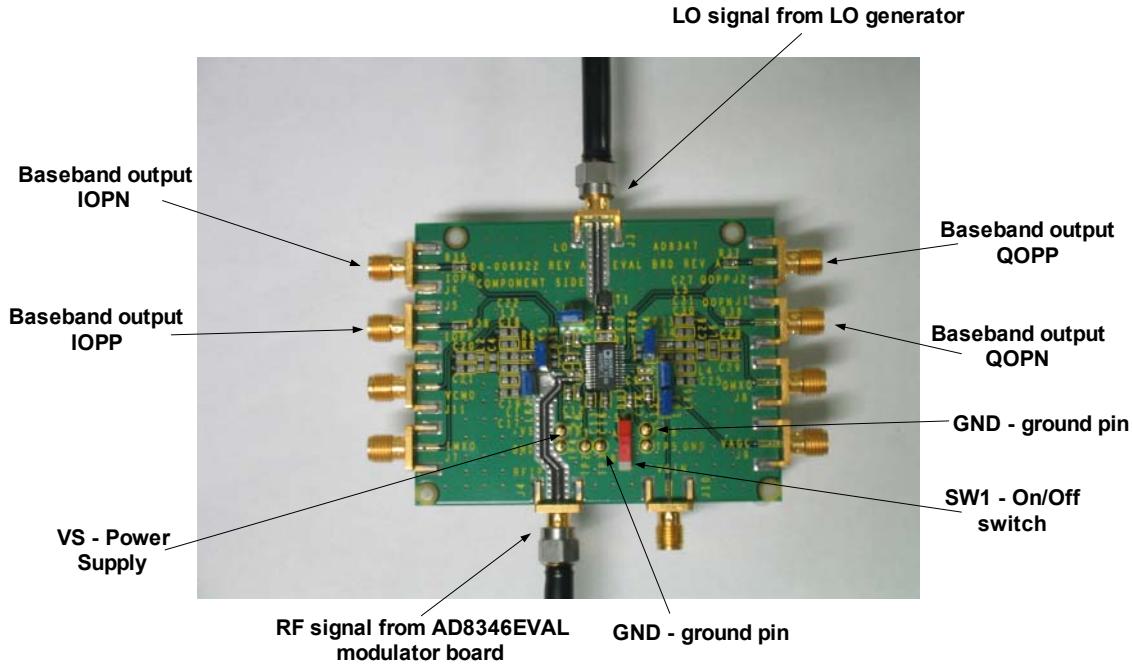


Figure 31. AD8347EVAL Cable/Signal Connections.

The LabVIEW control program is used generate the amplitude and phase of the RF output signal from the AD8346EVAL board. This RF signal is then sent to the RFIP of the AD8347EVAL board, as shown in Figure 31. Finally, the baseband output signals (IOPP, IOPN, QOPP and QOPN) are measured with respect to ground using a digital multi-meter.

4. Experimental Measurements

The test bench is configured for a 360° sweep in 5° increments using the LabVIEW control program. The results of the measured baseband output values are tabulated in Table 3.

TRANSMIT PHASE ($^\circ$)	IOPP (VDC) VOLTAGE	IOPN (VDC) VOLTAGE	QOPP (VDC) VOLTAGE	QOPN (VDC) VOLTAGE
0	1.0388	0.9907	1.1180	0.9105
5	1.0340	0.9958	1.1165	0.9120
10	1.0290	1.008	1.1144	0.9141
15	1.0232	1.0068	1.1114	0.9169
20	1.0165	1.0135	1.1076	0.9208
25	1.0015	1.0292	1.0979	0.9306
30	0.9450	1.0865	1.0420	0.9864
35	0.9248	1.1066	1.0028	1.0244
40	0.9213	1.1097	0.9956	1.0315
45	0.9193	1.1118	0.9908	1.0362
50	0.9181	1.1134	0.9869	1.0398
55	0.9169	1.1145	0.9831	1.0435
60	0.9159	1.1154	0.9797	1.0468
65	0.9152	1.1162	0.9768	1.0497
70	0.9146	1.1168	0.9733	1.0529
75	0.9142	1.1172	0.9702	1.0562
80	0.9139	1.1175	0.9676	1.0591
85	0.9136	1.1178	0.9642	1.0619
90	0.9134	1.1180	0.9605	1.0657
95	0.9136	1.1178	0.9563	1.0700
100	0.9142	1.1172	0.9513	1.0748

Table 3. Results of AD8347EVAL Bench Test.

TRANSMIT PHASE (°)	IOPP (VDC) VOLTAGE	IOPN (VDC) VOLTAGE	QOPP (VDC) VOLTAGE	QOPN (VDC) VOLTAGE
105	0.9153	1.1162	0.9465	1.0797
110	0.9172	1.1143	0.9399	1.0863
115	0.9247	1.1069	0.9131	1.1141
120	0.9815	1.0489	0.8322	1.1929
125	1.0107	1.0194	0.8138	1.2109
130	1.0174	1.0123	0.8096	1.2152
135	1.0218	1.0081	0.8070	1.2181
140	1.0255	1.0041	0.8048	1.2199
145	1.0292	1.0005	0.8026	1.2216
150	1.0320	0.9975	0.8011	1.2233
155	1.0351	0.9943	0.7998	1.2246
160	1.0383	0.9912	0.7990	1.2257
165	1.0407	0.9887	0.7980	1.2267
170	1.0436	0.9858	0.7972	1.2276
175	1.0466	0.9827	0.7962	1.2284
180	1.0496	0.9795	0.7954	1.2291
185	1.0541	0.9751	0.7948	1.2295
190	1.0587	0.9705	0.7944	1.2298
195	1.0631	0.9660	0.7946	1.2297
200	1.0688	0.9602	0.7953	1.2295
205	1.0866	0.9420	0.7972	1.2275
210	1.1898	0.8375	0.8508	1.1743
215	1.2355	0.7914	0.9218	1.1039
220	1.2405	0.7863	0.9390	1.0877
225	1.2422	0.7845	0.9437	1.0823
230	1.2433	0.7835	0.9473	1.0791

Table 3 (Continued). Results of AD8347EVAL Bench Test.

TRANSMIT PHASE (°)	IOPP (VDC) VOLTAGE	IOPN (VDC) VOLTAGE	QOPP (VDC) VOLTAGE	QOPN (VDC) VOLTAGE
235	1.2441	0.7826	0.9503	1.0760
240	1.2446	0.7820	0.9534	1.0728
245	1.2451	0.7815	0.9565	1.0697
250	1.2455	0.7811	0.9595	1.0671
255	1.2457	0.7809	0.9618	1.0643
260	1.2458	0.7806	0.9646	1.0616
265	1.2459	0.7807	0.9678	1.0588
270	1.2458	0.7806	0.9706	1.0557
275	1.2454	0.7811	0.9750	1.0512
280	1.2445	0.7821	0.9793	1.0471
285	1.2431	0.7836	0.9843	1.0419
290	1.2409	0.7858	0.9897	1.0367
295	1.2374	0.7896	0.9981	1.0290
300	1.2234	0.8037	1.0280	0.9977
305	1.1307	0.8986	1.1105	0.9160
310	1.0771	0.9523	1.1217	0.9069
315	1.0688	0.9603	1.1223	0.9063
320	1.0647	0.9645	1.1226	0.9060
325	1.0610	0.9684	1.1224	0.9059
330	1.0578	0.9714	1.1223	0.9060
335	1.0546	0.9748	1.1219	0.9064
340	1.0511	0.9781	1.1214	0.9069
345	1.0486	0.9807	1.1210	0.9073
350	1.0452	0.9841	1.1202	0.9080
355	1.0421	0.9872	1.1195	0.9086

Table 3 (Continued). Results of AD8347EVAL Bench Test.

After measuring the four baseband outputs, the next step is to calculate the differential voltages for I and Q , and then use Equation (13) to plot the received phase and compare it with the transmitted phase. The expected waveform should be a straight line with a slope of +1. A MATLAB program was written to plot and analyze the measured data.

The measured differential I and Q components are plotted against the transmitted phase and are shown in Figure 32. Ideally, the I and Q signals should be sinusoidal waveforms with zero dc level. The plotted I and Q signals resemble cosine and sine waves, respectively, but they have dc shift and their phases are offset. A manually derived dc offset value was introduced to the I and Q signal to bring the dc level to zero, and the result is also shown in Figure 32. The dc level of the I and Q signals were probably due to phase error in the transmitted RF signal from the AD8346EVAL board.

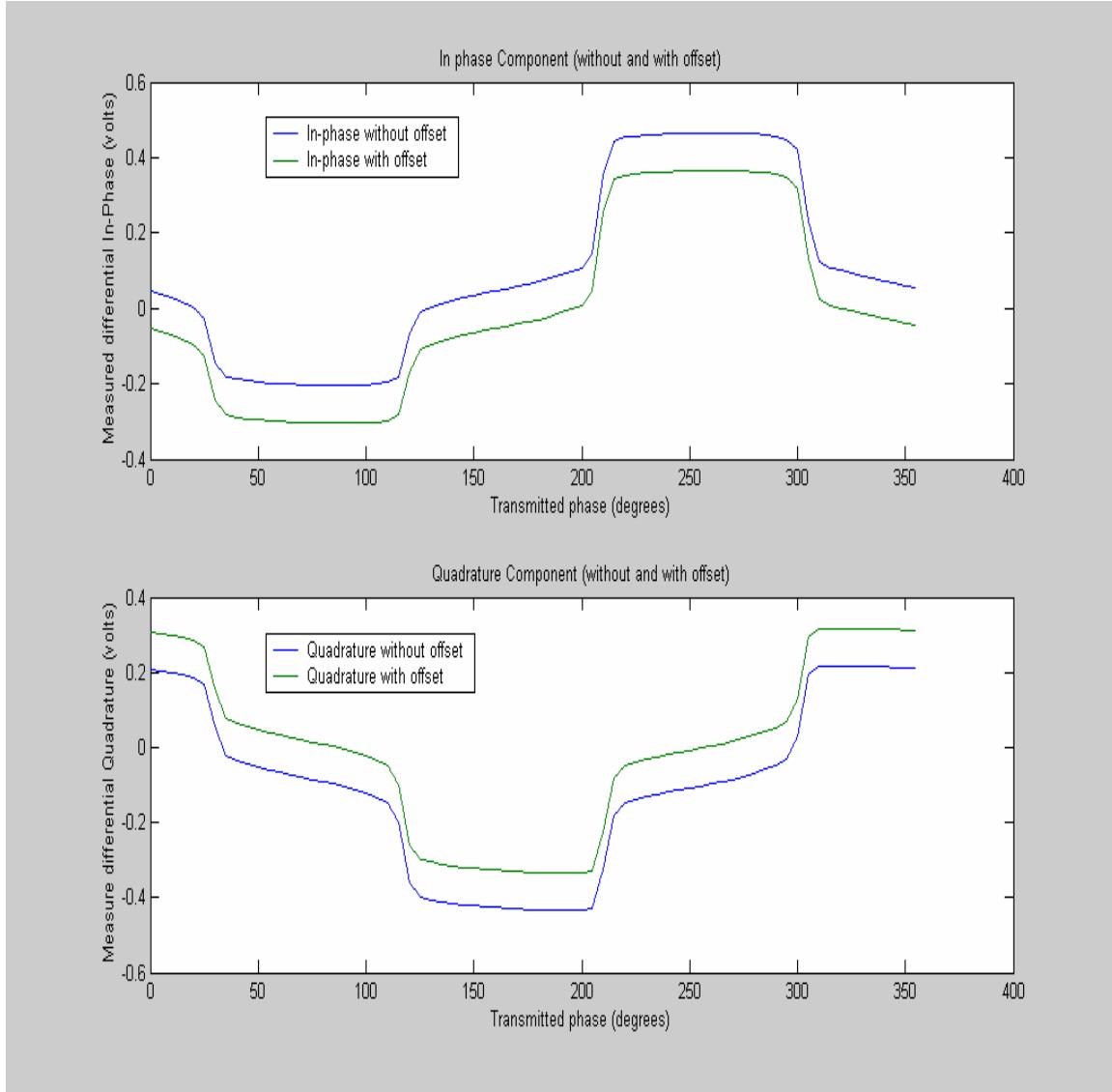


Figure 32. Measured Differential I and Q Components Without and With Offsets.

Figure 33 shows that the I and Q signals exhibit characteristics similar to the cosine and sine waveform, respectively, but with some phase offsets. A phase offset is not of concern, since any offset in the transmitted signal would be present in all elements of the array. Only the phase progression (differences in phase) affects the receive array output.

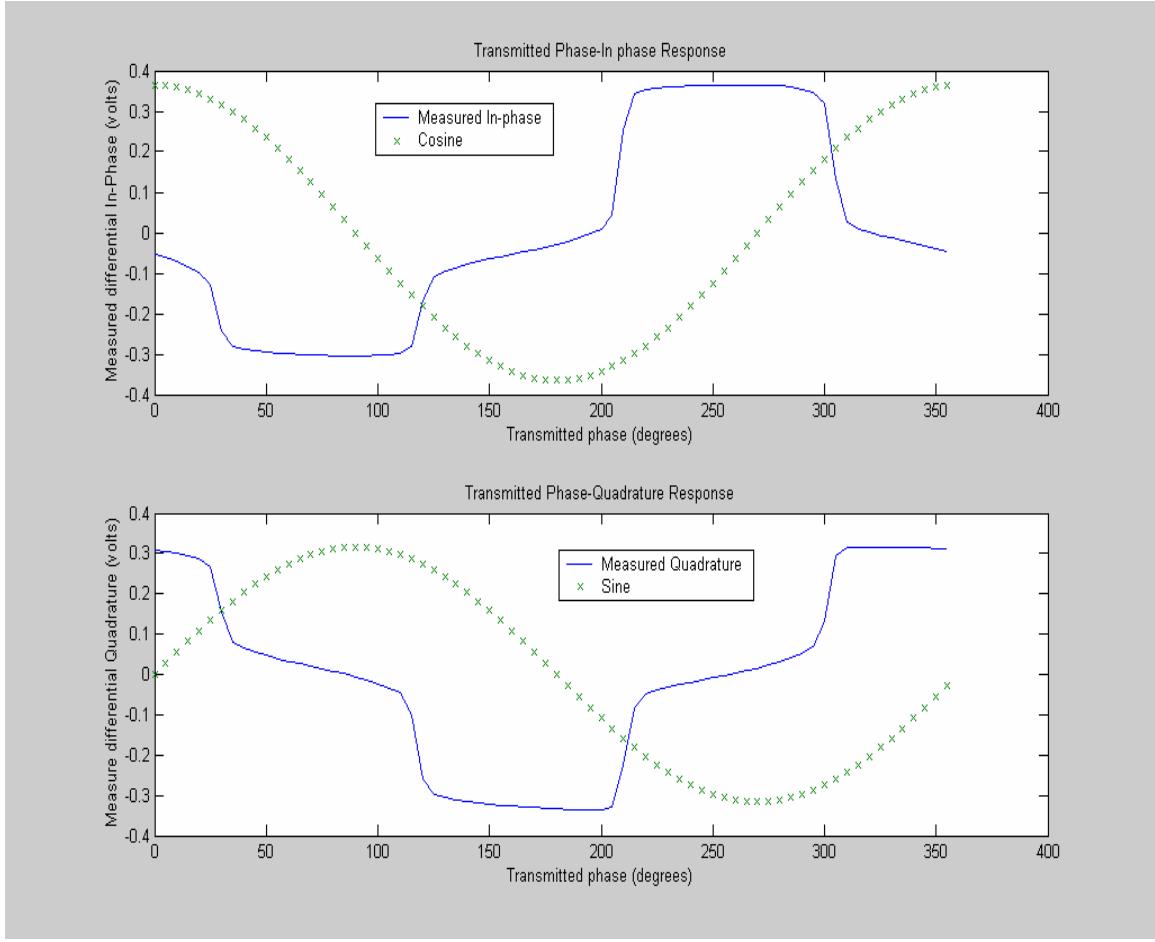


Figure 33. Measured Differential *I* and *Q* Components Versus Transmitted Phase Response.

Figure 34 shows the Transmitted Phase versus Measured Phase response of the AD8347EVAL board. The measured phase is fairly linear, but with some ripples. The ripples represent phase error. The ripple is due to measurement error and phase errors that occur in both the modulator used for the transmitting the signal, and the demodulator circuit. Esswein's measurements [3] showed that the modulator phase errors were greatest in the diagonals of the *I-Q* plane. For example, if the controlling software commands the modulator to a 45° phase shift, the actual phase shift might be 49.3° . This type of phase error is greatest at 45° , 135° , 225° , and 315° , which fits the period of the ripple in Figure 34.

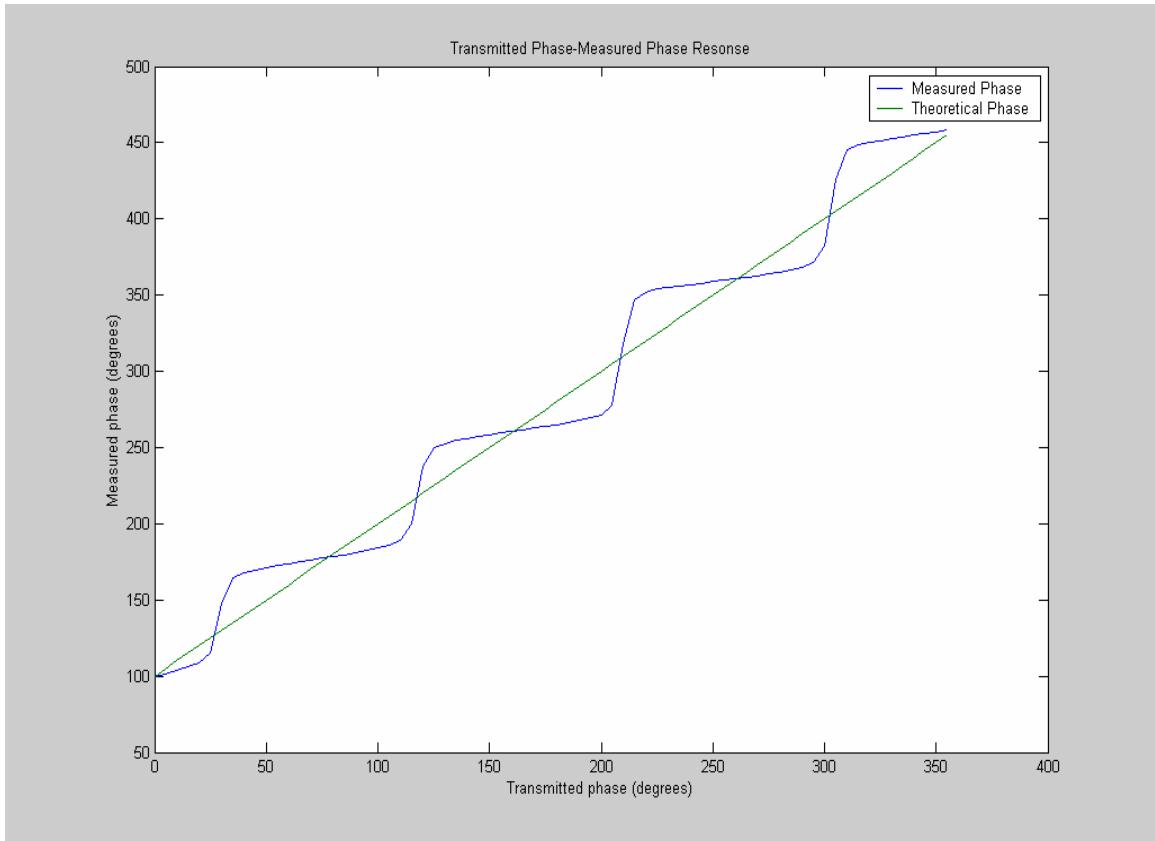


Figure 34. Transmitted Phase Versus Measured Phase Response.

C. SUMMARY

This chapter has presented the experimental setups of the AD8346EVAL modulator and AD8347EVAL demodulator boards. The results of the modulator board obtained from the experiment are incorporated into the simulation of a linear array and the resulted radiation patterns from the simulation showed that the modulator board does not have a wide instantaneous bandwidth characteristic. Also, the results of the demodulator board obtained from the experiment indicated that the board works successfully as a phase shifter.

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IV. CONCLUSIONS AND RECOMMENDATIONS

A. SUMMARY

Digital antennas have the potential of satisfying the requirements of many systems simultaneously. They are flexible, and capable of handling wide bandwidths, and can perform multiple functions. The bandwidth of the modulator and demodulator must match the bandwidth of the signal for efficient operation. Measurements on the AD8346EVAL transmit modulator revealed the instantaneous bandwidth is much narrower than the specified frequency range of operation (0.8 to 2.5 GHz). The effects of the phase slope and amplitude variations on the pattern of a linear array were determined by simulations that incorporated the measured data. The simulation showed unacceptable beam squint with frequency. Therefore, it appears that the AD8346EVAL modulator can only be used over a portion of the specified band. That is, the tunable bandwidth may be from 0.8 to 2.5 GHz, but the instantaneous bandwidth will be significantly less.

On the receive side, the AD8347EVAL demodulator board was successfully configured to operate as a phase shifter. Phase shifted transmit signals were injected into the demodulator, and corresponding phase shifts were measured via the *I* and *Q* voltages. A periodic phase error was identified and attributed to the phase error inherent in the modulator board.

B. RECOMMENDATIONS FOR FUTURE WORK

1. Arrays Transmit Antenna with Time Delay Unit

The next step in the transmit antenna research is to determine if some type of time delay unit can be applied so as to increase the instantaneous bandwidth of the modulator board. The time delay unit can be implemented either analog RF TDU or digital time delay filter.

Analog RF TDU is large, expensive, and is likely to introduce quantization error and band edge phase error [9]. On the other hand, a digital time delay filter may consist of an ADC, a digital delay filter, and DAC [10]. All these components are small and it is

more adaptable to the changing phase that is required in each element since it is usually implemented in software. However, software TDUs are difficult to implement on the transmit antenna. The waveform into the modulator would have to be modified so that the waveform spectrum multiplied by the frequency characteristic of the modulator gives the desired broadband performance.

2. Transmit Antenna with Amplitude Equalizers

The measured frequency-magnitude response plot of the AD8346EVAL board (Figures 17) showed that the magnitude is not constant. Therefore, some equalization process is required to obtain constant magnitude across all of the operating frequencies.

3. Receive Antenna

The next step in the receive antenna research is to integrate the AD8347EVAL board with a compatible ADC. The ADC could then connect to a computer with digital processing capability to form the receive beams.

A 3-D phased array receive antenna could then be built with the necessary I/O boards to allow the GA to be verified for the entire beam forming process from transmission to reception as shown in Figure 28.

APPENDIX A: GLOSSARY OF TERMINOLOGY

3-D	Three Dimensional
AD	Analog Devices
ADC	Analog-to-Digital-Converter
AF	Array Factor
AGC	Auto Gain Control
CDMA	Code Division Multiple Access
COTS	Commercial Off The Shelf
GA	Genetic Algorithm
GHz	Giga Hertz
GSM	Global Satellite Mobile
I/O	Input/Output
IR	Infrared
LCDR	Lieutenant Commander
LO	Local Oscillator
NI	National Instruments
RCS	Radar Cross Section
RF	Radio Frequency
RFIC	Radio Frequency Integrated Circuit
TDSI	Temasek Defense Systems Institute
TDU	Time Delay Unit
UHF/VHF	Ultra High Frequency/Very High Frequency
VNA	Vector Network Analyzer

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APPENDIX B: MATLAB CODE OF THE PHASED ARRAY TRANSMIT ANTENNA FOR GENERATING FIGURES 17 TO 20

```
%Analysis of the measured data for the phased array transmit antenna.
%The S21 parameters are loaded to calculate the amplitudes and phases of the AD8346EVAL board.
%The amplitudes and phases versus frequency response are plotted in Figure 17 to 20.
```

```
clear
load Test_0.m, load Test_15.m, load Test_30.m, load Test_45.m, load Test_60.m;
load Test_75.m, load Test_90.m, load Test_105.m, load Test_120.m, load Test_135.m;
load Test_150.m, load Test_165.m, load Test_180.m, load Test_195.m, load Test_210.m;
load Test_225.m, load Test_240.m, load Test_255.m, load Test_270.m, load Test_285.m;
load Test_300.m, load Test_315.m, load Test_330.m, load Test_345.m;

%Frequency sweep from 0.8 to 2.5 GHz with 201 samplers.
fstart = 0.8;
fstop = 2.5;
nf = 201;
fstep = (fstop - fstart)/(nf - 1);
f = [fstart:fstep:fstop];

%Convert the data to real and imaginary parts.
I0 = Test_0(:,1);
Q0 = Test_0(:,2);
V0 = I0 + j*Q0;

I15 = Test_15(:,1);
Q15 = Test_15(:,2);
V15 = I15 + j*Q15;

I30 = Test_30(:,1);
Q30 = Test_30(:,2);
V30 = I30 + j*Q30;

I45 = Test_45(:,1);
Q45 = Test_45(:,2);
V45 = I45 + j*Q45;

I60 = Test_60(:,1);
Q60 = Test_60(:,2);
V60 = I60 + j*Q60;

I75 = Test_75(:,1);
Q75 = Test_75(:,2);
V75 = I75 + j*Q75;

I90 = Test_90(:,1);
Q90 = Test_90(:,2);
V90 = I90 + j*Q90;

I105 = Test_105(:,1);
Q105 = Test_105(:,2);
V105 = I105 + j*Q105;

I120 = Test_120(:,1);
Q120 = Test_120(:,2);
V120 = I120 + j*Q120;
```

```

I135 = Test_135(:,1);
Q135 = Test_135(:,2);
V135 = I135 + j*Q135;

I150 = Test_150(:,1);
Q150 = Test_150(:,2);
V150 = I150 + j*Q150;

I165 = Test_165(:,1);
Q165 = Test_165(:,2);
V165 = I165 + j*Q165;

I180 = Test_180(:,1);
Q180 = Test_180(:,2);
V180 = I180 + j*Q180;

I195 = Test_195(:,1);
Q195 = Test_195(:,2);
V195 = I195 + j*Q195;

I210 = Test_210(:,1);
Q210 = Test_210(:,2);
V210 = I210 + j*Q210;

I225 = Test_225(:,1);
Q225 = Test_225(:,2);
V225 = I225 + j*Q225;

I240 = Test_240(:,1);
Q240 = Test_240(:,2);
V240 = I240 + j*Q240;

I255 = Test_255(:,1);
Q255 = Test_255(:,2);
V255 = I255 + j*Q255;

I270 = Test_270(:,1);
Q270 = Test_270(:,2);
V270 = I270 + j*Q270;

I285 = Test_285(:,1);
Q285 = Test_285(:,2);
V285 = I285 + j*Q285;

I300 = Test_300(:,1);
Q300 = Test_300(:,2);
V300 = I300 + j*Q300;

I315 = Test_315(:,1);
Q315 = Test_315(:,2);
V315 = I315 + j*Q315;

I330 = Test_330(:,1);
Q330 = Test_330(:,2);
V330 = I330 + j*Q330;

I345 = Test_345(:,1);
Q345 = Test_345(:,2);
V345 = I345 + j*Q345;

%Convert to magnitude and phase.
Mag0 = 10*log(abs(V0)), Phase0 = unwrap(angle(V0))*(180/pi);

```

```

Mag15 = 10*log(abs(V15)), Phase15 = unwrap(angle(V15))*(180/pi);
Mag30 = 10*log(abs(V30)), Phase30 = unwrap(angle(V30))*(180/pi);
Mag45 = 10*log(abs(V45)), Phase45 = unwrap(angle(V45))*(180/pi);
Mag60 = 10*log(abs(V60)), Phase60 = unwrap(angle(V60))*(180/pi);
Mag75 = 10*log(abs(V75)), Phase75 = unwrap(angle(V75))*(180/pi);
Mag90 = 10*log(abs(V90)), Phase90 = unwrap(angle(V90))*(180/pi);
Mag105 = 10*log(abs(V105)), Phase105 = unwrap(angle(V105))*(180/pi);
Mag120 = 10*log(abs(V120)), Phase120 = unwrap(angle(V120))*(180/pi);
Mag135 = 10*log(abs(V135)), Phase135 = unwrap(angle(V135))*(180/pi);
Mag150 = 10*log(abs(V150)), Phase150 = unwrap(angle(V150))*(180/pi);
Mag165 = 10*log(abs(V165)), Phase165 = unwrap(angle(V165))*(180/pi);
Mag180 = 10*log(abs(V180)), Phase180 = unwrap(angle(V180))*(180/pi);
Mag195 = 10*log(abs(V195)), Phase195 = unwrap(angle(V195))*(180/pi);
Mag210 = 10*log(abs(V210)), Phase210 = unwrap(angle(V210))*(180/pi);
Mag225 = 10*log(abs(V225)), Phase225 = unwrap(angle(V225))*(180/pi);
Mag240 = 10*log(abs(V240)), Phase240 = unwrap(angle(V240))*(180/pi);
Mag255 = 10*log(abs(V255)), Phase255 = unwrap(angle(V255))*(180/pi);
Mag270 = 10*log(abs(V270)), Phase270 = unwrap(angle(V270))*(180/pi);
Mag285 = 10*log(abs(V285)), Phase285 = unwrap(angle(V285))*(180/pi);
Mag300 = 10*log(abs(V300)), Phase300 = unwrap(angle(V300))*(180/pi);
Mag315 = 10*log(abs(V315)), Phase315 = unwrap(angle(V315))*(180/pi);
Mag330 = 10*log(abs(V330)), Phase330 = unwrap(angle(V330))*(180/pi);
Mag345 = 10*log(abs(V345)), Phase345 = unwrap(angle(V345))*(180/pi);

subplot(211)
%figure(1)
plot(f, Mag0, f, Mag15, f, Mag30, f, Mag45, f, Mag60, f, Mag75)
legend('0 degree','15 degrees','30 degrees','45 degrees','60 degrees','75 degrees');
xlabel('Frequency (GHz)');
ylabel('Magnitude (dB)');
title('Frequency-Magnitude Response');
%print -depsc -tiff -r300 magnitude1

subplot(212)
%figure(2)
plot(f, Phase0, f, Phase15, f, Phase30, f, Phase45, f, Phase60, f, Phase75)
legend('0 degree','15 degrees','30 degrees','45 degrees','60 degrees','75 degrees');
xlabel('Frequency (GHz)')
ylabel('Phase (degrees)')
title('Frequency-Phase Response')
print -depsc -tiff -r300 mag_phase1

```

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APPENDIX C: MATHCAD CODE FOR SIMULATING THE RADIATION PATTERN OF THE LINEAR PHASED ARRAY

We will assume a N -element linear array with uniformly-spaced identical elements. We will assume that all is happening in the xz -plane.

X := 201

Y := 2

`x := 0.. X - 1`

$y := 0..Y - 1$

$$j := \sqrt{-1}$$

Frequency Range: 0.8 - 2.5 GHz with 201 data points.

$$f_{\text{start}} := 0.8$$

$$f_{stop} := 2.5$$

$$\Delta f := \frac{(f_{\text{stop}} - f_{\text{start}})}{X - 1}$$

$$\Delta f = 8.5 \times 10^{-3}$$

freq := x·Δf + f_{start}

-X start
Reading Transmitter's data files. Total 360 files at 1 degree increment

Deg0:=READPRN("Test_0.txt")

Deg1:= READPRN("Test_1.txt")

1

1

1

Deg358:= READPRN("Test 358.txt")

Deg359:= READPRN("Test 359.txt")

Defining the vectors. Converting to real and imaginary parts for 360 measured data.

Vector_0_x := Deg0_{x, 0} + j.Deg0_{x, 1}
 Vector_1_x := Deg1_{x, 0} + j.Deg1_{x, 1}
 Vector_2_x := Deg2_{x, 0} + j.Deg2_{x, 1}
 Vector_3_x := Deg3_{x, 0} + j.Deg3_{x, 1}
 Vector_4_x := Deg4_{x, 0} + j.Deg4_{x, 1}
 Vector_5_x := Deg5_{x, 0} + j.Deg5_{x, 1}
 Vector_6_x := Deg6_{x, 0} + j.Deg6_{x, 1}
 Vector_7_x := Deg7_{x, 0} + j.Deg7_{x, 1}
 Vector_8_x := Deg8_{x, 0} + j.Deg8_{x, 1}
 Vector_9_x := Deg9_{x, 0} + j.Deg9_{x, 1}
 Vector_10_x := Deg10_{x, 0} + j.Deg10_{x,}
 Vector_11_x := Deg11_{x, 0} + j.Deg11_{x,}

```

Vector_181x := Deg181x, 0 + j.Deg181x, 1
Vector_182x := Deg182x, 0 + j.Deg182x, 1
Vector_183x := Deg183x, 0 + j.Deg183x, 1
Vector_184x := Deg184x, 0 + j.Deg184x, 1
Vector_185x := Deg185x, 0 + j.Deg185x, 1
Vector_186x := Deg186x, 0 + j.Deg186x, 1
Vector_187x := Deg187x, 0 + j.Deg187x, 1
Vector_188x := Deg188x, 0 + j.Deg188x, 1
Vector_189x := Deg189x, 0 + j.Deg189x, 1
Vector_190x := Deg190x, 0 + j.Deg190x, 1
Vector_191x := Deg191x, 0 + j.Deg191x, 1
Vector_192x := Deg192x, 0 + j.Deg192x, 1

```

```

Vector_12_x := Deg12_x,0 + j·Deg12_x,1
Vector_13_x := Deg13_x,0 + j·Deg13_x,1
Vector_14_x := Deg14_x,0 + j·Deg14_x,1
Vector_15_x := Deg15_x,0 + j·Deg15_x,1
Vector_16_x := Deg16_x,0 + j·Deg16_x,1
Vector_17_x := Deg17_x,0 + j·Deg17_x,1
Vector_18_x := Deg18_x,0 + j·Deg18_x,1
Vector_19_x := Deg19_x,0 + j·Deg19_x,1
Vector_20_x := Deg20_x,0 + j·Deg20_x,1
Vector_21_x := Deg21_x,0 + j·Deg21_x,1
Vector_22_x := Deg22_x,0 + j·Deg22_x,1
Vector_23_x := Deg23_x,0 + j·Deg23_x,1
Vector_24_x := Deg24_x,0 + j·Deg24_x,1
Vector_25_x := Deg25_x,0 + j·Deg25_x,1
Vector_26_x := Deg26_x,0 + j·Deg26_x,1
Vector_27_x := Deg27_x,0 + j·Deg27_x,1
Vector_28_x := Deg28_x,0 + j·Deg28_x,1
Vector_29_x := Deg29_x,0 + j·Deg29_x,1
Vector_30_x := Deg30_x,0 + j·Deg30_x,1
Vector_31_x := Deg31_x,0 + j·Deg31_x,1
Vector_32_x := Deg32_x,0 + j·Deg32_x,1
Vector_33_x := Deg33_x,0 + j·Deg33_x,1
Vector_34_x := Deg34_x,0 + j·Deg34_x,1
Vector_35_x := Deg35_x,0 + j·Deg35_x,1
Vector_36_x := Deg36_x,0 + j·Deg36_x,1
Vector_37_x := Deg37_x,0 + j·Deg37_x,1
Vector_38_x := Deg38_x,0 + j·Deg38_x,1
Vector_39_x := Deg39_x,0 + j·Deg39_x,1
Vector_40_x := Deg40_x,0 + j·Deg40_x,1
Vector_41_x := Deg41_x,0 + j·Deg41_x,1
Vector_42_x := Deg42_x,0 + j·Deg42_x,1
Vector_43_x := Deg43_x,0 + j·Deg43_x,1
Vector_44_x := Deg44_x,0 + j·Deg44_x,1
Vector_45_x := Deg45_x,0 + j·Deg45_x,1
Vector_46_x := Deg46_x,0 + j·Deg46_x,1
Vector_47_x := Deg47_x,0 + j·Deg47_x,1
Vector_193_x := Deg193_x,0 + j·Deg193_x,1
Vector_194_x := Deg194_x,0 + j·Deg194_x,1
Vector_195_x := Deg195_x,0 + j·Deg195_x,1
Vector_196_x := Deg196_x,0 + j·Deg196_x,1
Vector_197_x := Deg197_x,0 + j·Deg197_x,1
Vector_198_x := Deg198_x,0 + j·Deg198_x,1
Vector_199_x := Deg199_x,0 + j·Deg199_x,1
Vector_200_x := Deg200_x,0 + j·Deg200_x,1
Vector_201_x := Deg201_x,0 + j·Deg201_x,1
Vector_202_x := Deg202_x,0 + j·Deg202_x,1
Vector_203_x := Deg203_x,0 + j·Deg203_x,1
Vector_204_x := Deg204_x,0 + j·Deg204_x,1
Vector_205_x := Deg205_x,0 + j·Deg205_x,1
Vector_206_x := Deg206_x,0 + j·Deg206_x,1
Vector_207_x := Deg207_x,0 + j·Deg207_x,1
Vector_208_x := Deg208_x,0 + j·Deg208_x,1
Vector_209_x := Deg209_x,0 + j·Deg209_x,1
Vector_210_x := Deg210_x,0 + j·Deg210_x,1
Vector_211_x := Deg211_x,0 + j·Deg211_x,1
Vector_212_x := Deg212_x,0 + j·Deg212_x,1
Vector_213_x := Deg213_x,0 + j·Deg213_x,1
Vector_214_x := Deg214_x,0 + j·Deg214_x,1
Vector_215_x := Deg215_x,0 + j·Deg215_x,1
Vector_216_x := Deg216_x,0 + j·Deg216_x,1
Vector_217_x := Deg217_x,0 + j·Deg217_x,1
Vector_218_x := Deg218_x,0 + j·Deg218_x,1
Vector_219_x := Deg219_x,0 + j·Deg219_x,1
Vector_220_x := Deg220_x,0 + j·Deg220_x,1
Vector_221_x := Deg221_x,0 + j·Deg221_x,1
Vector_222_x := Deg222_x,0 + j·Deg222_x,1
Vector_223_x := Deg223_x,0 + j·Deg223_x,1
Vector_224_x := Deg224_x,0 + j·Deg224_x,1
Vector_225_x := Deg225_x,0 + j·Deg225_x,1
Vector_226_x := Deg226_x,0 + j·Deg226_x,1
Vector_227_x := Deg227_x,0 + j·Deg227_x,1
Vector_228_x := Deg228_x,0 + j·Deg228_x,1

```

Vector_48 _x := Deg48 _{x, 0} + j·Deg48 _{x, 1}	Vector_229 _x := Deg229 _{x, 0} + j·Deg229 _{x, 1}
Vector_49 _x := Deg49 _{x, 0} + j·Deg49 _{x, 1}	Vector_230 _x := Deg230 _{x, 0} + j·Deg230 _{x, 1}
Vector_50 _x := Deg50 _{x, 0} + j·Deg50 _{x, 1}	Vector_231 _x := Deg231 _{x, 0} + j·Deg231 _{x, 1}
Vector_51 _x := Deg51 _{x, 0} + j·Deg51 _{x, 1}	Vector_232 _x := Deg232 _{x, 0} + j·Deg232 _{x, 1}
Vector_52 _x := Deg52 _{x, 0} + j·Deg52 _{x, 1}	Vector_233 _x := Deg233 _{x, 0} + j·Deg233 _{x, 1}
Vector_53 _x := Deg53 _{x, 0} + j·Deg53 _{x, 1}	Vector_234 _x := Deg234 _{x, 0} + j·Deg234 _{x, 1}
Vector_54 _x := Deg54 _{x, 0} + j·Deg54 _{x, 1}	Vector_235 _x := Deg235 _{x, 0} + j·Deg235 _{x, 1}
Vector_55 _x := Deg55 _{x, 0} + j·Deg55 _{x, 1}	Vector_236 _x := Deg236 _{x, 0} + j·Deg236 _{x, 1}
Vector_56 _x := Deg56 _{x, 0} + j·Deg56 _{x, 1}	Vector_237 _x := Deg237 _{x, 0} + j·Deg237 _{x, 1}
Vector_57 _x := Deg57 _{x, 0} + j·Deg57 _{x, 1}	Vector_238 _x := Deg238 _{x, 0} + j·Deg238 _{x, 1}
Vector_58 _x := Deg58 _{x, 0} + j·Deg58 _{x, 1}	Vector_239 _x := Deg239 _{x, 0} + j·Deg239 _{x, 1}
Vector_59 _x := Deg59 _{x, 0} + j·Deg59 _{x, 1}	Vector_240 _x := Deg240 _{x, 0} + j·Deg240 _{x, 1}
Vector_60 _x := Deg60 _{x, 0} + j·Deg60 _{x, 1}	Vector_241 _x := Deg241 _{x, 0} + j·Deg241 _{x, 1}
Vector_61 _x := Deg61 _{x, 0} + j·Deg61 _{x, 1}	Vector_242 _x := Deg242 _{x, 0} + j·Deg242 _{x, 1}
Vector_62 _x := Deg62 _{x, 0} + j·Deg62 _{x, 1}	Vector_243 _x := Deg243 _{x, 0} + j·Deg243 _{x, 1}
Vector_63 _x := Deg63 _{x, 0} + j·Deg63 _{x, 1}	Vector_244 _x := Deg244 _{x, 0} + j·Deg244 _{x, 1}
Vector_64 _x := Deg64 _{x, 0} + j·Deg64 _{x, 1}	Vector_245 _x := Deg245 _{x, 0} + j·Deg245 _{x, 1}
Vector_65 _x := Deg65 _{x, 0} + j·Deg65 _{x, 1}	Vector_246 _x := Deg246 _{x, 0} + j·Deg246 _{x, 1}
Vector_66 _x := Deg66 _{x, 0} + j·Deg66 _{x, 1}	Vector_247 _x := Deg247 _{x, 0} + j·Deg247 _{x, 1}
Vector_67 _x := Deg67 _{x, 0} + j·Deg67 _{x, 1}	Vector_248 _x := Deg248 _{x, 0} + j·Deg248 _{x, 1}
Vector_68 _x := Deg68 _{x, 0} + j·Deg68 _{x, 1}	Vector_249 _x := Deg249 _{x, 0} + j·Deg249 _{x, 1}
Vector_69 _x := Deg69 _{x, 0} + j·Deg69 _{x, 1}	Vector_250 _x := Deg250 _{x, 0} + j·Deg250 _{x, 1}
Vector_70 _x := Deg70 _{x, 0} + j·Deg70 _{x, 1}	Vector_251 _x := Deg251 _{x, 0} + j·Deg251 _{x, 1}
Vector_71 _x := Deg71 _{x, 0} + j·Deg71 _{x, 1}	Vector_252 _x := Deg252 _{x, 0} + j·Deg252 _{x, 1}
Vector_72 _x := Deg72 _{x, 0} + j·Deg72 _{x, 1}	Vector_253 _x := Deg253 _{x, 0} + j·Deg253 _{x, 1}
Vector_73 _x := Deg73 _{x, 0} + j·Deg73 _{x, 1}	Vector_254 _x := Deg254 _{x, 0} + j·Deg254 _{x, 1}
Vector_74 _x := Deg74 _{x, 0} + j·Deg74 _{x, 1}	Vector_255 _x := Deg255 _{x, 0} + j·Deg255 _{x, 1}
Vector_75 _x := Deg75 _{x, 0} + j·Deg75 _{x, 1}	Vector_256 _x := Deg256 _{x, 0} + j·Deg256 _{x, 1}
Vector_76 _x := Deg76 _{x, 0} + j·Deg76 _{x, 1}	Vector_257 _x := Deg257 _{x, 0} + j·Deg257 _{x, 1}
Vector_77 _x := Deg77 _{x, 0} + j·Deg77 _{x, 1}	Vector_258 _x := Deg258 _{x, 0} + j·Deg258 _{x, 1}
Vector_78 _x := Deg78 _{x, 0} + j·Deg78 _{x, 1}	Vector_259 _x := Deg259 _{x, 0} + j·Deg259 _{x, 1}
Vector_79 _x := Deg79 _{x, 0} + j·Deg79 _{x, 1}	Vector_260 _x := Deg260 _{x, 0} + j·Deg260 _{x, 1}
Vector_80 _x := Deg80 _{x, 0} + j·Deg80 _{x, 1}	Vector_261 _x := Deg261 _{x, 0} + j·Deg261 _{x, 1}
Vector_81 _x := Deg81 _{x, 0} + j·Deg81 _{x, 1}	Vector_262 _x := Deg262 _{x, 0} + j·Deg262 _{x, 1}
Vector_82 _x := Deg82 _{x, 0} + j·Deg82 _{x, 1}	Vector_263 _x := Deg263 _{x, 0} + j·Deg263 _{x, 1}
Vector_83 _x := Deg83 _{x, 0} + j·Deg83 _{x, 1}	Vector_264 _x := Deg264 _{x, 0} + j·Deg264 _{x, 1}

Vector_84 _x := Deg84 _{x, 0} + j·Deg84 _{x, 1}	Vector_265 _x := Deg265 _{x, 0} + j·Deg265 _{x, 1}
Vector_85 _x := Deg85 _{x, 0} + j·Deg85 _{x, 1}	Vector_266 _x := Deg266 _{x, 0} + j·Deg266 _{x, 1}
Vector_86 _x := Deg86 _{x, 0} + j·Deg86 _{x, 1}	Vector_267 _x := Deg267 _{x, 0} + j·Deg267 _{x, 1}
Vector_87 _x := Deg87 _{x, 0} + j·Deg87 _{x, 1}	Vector_268 _x := Deg268 _{x, 0} + j·Deg268 _{x, 1}
Vector_88 _x := Deg88 _{x, 0} + j·Deg88 _{x, 1}	Vector_269 _x := Deg269 _{x, 0} + j·Deg269 _{x, 1}
Vector_89 _x := Deg89 _{x, 0} + j·Deg89 _{x, 1}	Vector_270 _x := Deg270 _{x, 0} + j·Deg270 _{x, 1}
Vector_90 _x := Deg90 _{x, 0} + j·Deg90 _{x, 1}	Vector_271 _x := Deg271 _{x, 0} + j·Deg271 _{x, 1}
Vector_91 _x := Deg91 _{x, 0} + j·Deg91 _{x, 1}	Vector_272 _x := Deg272 _{x, 0} + j·Deg272 _{x, 1}
Vector_92 _x := Deg92 _{x, 0} + j·Deg92 _{x, 1}	Vector_273 _x := Deg273 _{x, 0} + j·Deg273 _{x, 1}
Vector_93 _x := Deg93 _{x, 0} + j·Deg93 _{x, 1}	Vector_274 _x := Deg274 _{x, 0} + j·Deg274 _{x, 1}
Vector_94 _x := Deg94 _{x, 0} + j·Deg94 _{x, 1}	Vector_275 _x := Deg275 _{x, 0} + j·Deg275 _{x, 1}
Vector_95 _x := Deg95 _{x, 0} + j·Deg95 _{x, 1}	Vector_276 _x := Deg276 _{x, 0} + j·Deg276 _{x, 1}
Vector_96 _x := Deg96 _{x, 0} + j·Deg96 _{x, 1}	Vector_277 _x := Deg277 _{x, 0} + j·Deg277 _{x, 1}
Vector_97 _x := Deg97 _{x, 0} + j·Deg97 _{x, 1}	Vector_278 _x := Deg278 _{x, 0} + j·Deg278 _{x, 1}
Vector_98 _x := Deg98 _{x, 0} + j·Deg98 _{x, 1}	Vector_279 _x := Deg279 _{x, 0} + j·Deg279 _{x, 1}
Vector_99 _x := Deg99 _{x, 0} + j·Deg99 _{x, 1}	Vector_280 _x := Deg280 _{x, 0} + j·Deg280 _{x, 1}
Vector_100 _x := Deg100 _{x, 0} + j·Deg100 _{x, 1}	Vector_281 _x := Deg281 _{x, 0} + j·Deg281 _{x, 1}
Vector_101 _x := Deg101 _{x, 0} + j·Deg101 _{x, 1}	Vector_282 _x := Deg282 _{x, 0} + j·Deg282 _{x, 1}
Vector_102 _x := Deg102 _{x, 0} + j·Deg102 _{x, 1}	Vector_283 _x := Deg283 _{x, 0} + j·Deg283 _{x, 1}
Vector_103 _x := Deg103 _{x, 0} + j·Deg103 _{x, 1}	Vector_284 _x := Deg284 _{x, 0} + j·Deg284 _{x, 1}
Vector_104 _x := Deg104 _{x, 0} + j·Deg104 _{x, 1}	Vector_285 _x := Deg285 _{x, 0} + j·Deg285 _{x, 1}
Vector_105 _x := Deg105 _{x, 0} + j·Deg105 _{x, 1}	Vector_286 _x := Deg286 _{x, 0} + j·Deg286 _{x, 1}
Vector_106 _x := Deg106 _{x, 0} + j·Deg106 _{x, 1}	Vector_287 _x := Deg287 _{x, 0} + j·Deg287 _{x, 1}
Vector_107 _x := Deg107 _{x, 0} + j·Deg107 _{x, 1}	Vector_288 _x := Deg288 _{x, 0} + j·Deg288 _{x, 1}
Vector_108 _x := Deg108 _{x, 0} + j·Deg108 _{x, 1}	Vector_289 _x := Deg289 _{x, 0} + j·Deg289 _{x, 1}
Vector_109 _x := Deg109 _{x, 0} + j·Deg109 _{x, 1}	Vector_290 _x := Deg290 _{x, 0} + j·Deg290 _{x, 1}
Vector_110 _x := Deg110 _{x, 0} + j·Deg110 _{x, 1}	Vector_291 _x := Deg291 _{x, 0} + j·Deg291 _{x, 1}
Vector_111 _x := Deg111 _{x, 0} + j·Deg111 _{x, 1}	Vector_292 _x := Deg292 _{x, 0} + j·Deg292 _{x, 1}
Vector_112 _x := Deg112 _{x, 0} + j·Deg112 _{x, 1}	Vector_293 _x := Deg293 _{x, 0} + j·Deg293 _{x, 1}
Vector_113 _x := Deg113 _{x, 0} + j·Deg113 _{x, 1}	Vector_294 _x := Deg294 _{x, 0} + j·Deg294 _{x, 1}
Vector_114 _x := Deg114 _{x, 0} + j·Deg114 _{x, 1}	Vector_295 _x := Deg295 _{x, 0} + j·Deg295 _{x, 1}
Vector_115 _x := Deg115 _{x, 0} + j·Deg115 _{x, 1}	Vector_296 _x := Deg296 _{x, 0} + j·Deg296 _{x, 1}
Vector_116 _x := Deg116 _{x, 0} + j·Deg116 _{x, 1}	Vector_297 _x := Deg297 _{x, 0} + j·Deg297 _{x, 1}
Vector_117 _x := Deg117 _{x, 0} + j·Deg117 _{x, 1}	Vector_298 _x := Deg298 _{x, 0} + j·Deg298 _{x, 1}
Vector_118 _x := Deg118 _{x, 0} + j·Deg118 _{x, 1}	Vector_299 _x := Deg299 _{x, 0} + j·Deg299 _{x, 1}
Vector_119 _x := Deg119 _{x, 0} + j·Deg119 _{x, 1}	Vector_300 _x := Deg300 _{x, 0} + j·Deg300 _{x, 1}

Vector_120 _x := Deg120 _{x, 0} + j·Deg120 _{x, 1}	Vector_301 _x := Deg301 _{x, 0} + j·Deg301 _{x, 1}
Vector_121 _x := Deg121 _{x, 0} + j·Deg121 _{x, 1}	Vector_302 _x := Deg302 _{x, 0} + j·Deg302 _{x, 1}
Vector_122 _x := Deg122 _{x, 0} + j·Deg122 _{x, 1}	Vector_303 _x := Deg303 _{x, 0} + j·Deg303 _{x, 1}
Vector_123 _x := Deg123 _{x, 0} + j·Deg123 _{x, 1}	Vector_304 _x := Deg304 _{x, 0} + j·Deg304 _{x, 1}
Vector_124 _x := Deg124 _{x, 0} + j·Deg124 _{x, 1}	Vector_305 _x := Deg305 _{x, 0} + j·Deg305 _{x, 1}
Vector_125 _x := Deg125 _{x, 0} + j·Deg125 _{x, 1}	Vector_306 _x := Deg306 _{x, 0} + j·Deg306 _{x, 1}
Vector_126 _x := Deg126 _{x, 0} + j·Deg126 _{x, 1}	Vector_307 _x := Deg307 _{x, 0} + j·Deg307 _{x, 1}
Vector_127 _x := Deg127 _{x, 0} + j·Deg127 _{x, 1}	Vector_308 _x := Deg308 _{x, 0} + j·Deg308 _{x, 1}
Vector_128 _x := Deg128 _{x, 0} + j·Deg128 _{x, 1}	Vector_309 _x := Deg309 _{x, 0} + j·Deg309 _{x, 1}
Vector_129 _x := Deg129 _{x, 0} + j·Deg129 _{x, 1}	Vector_310 _x := Deg310 _{x, 0} + j·Deg310 _{x, 1}
Vector_130 _x := Deg130 _{x, 0} + j·Deg130 _{x, 1}	Vector_311 _x := Deg311 _{x, 0} + j·Deg311 _{x, 1}
Vector_131 _x := Deg131 _{x, 0} + j·Deg131 _{x, 1}	Vector_312 _x := Deg312 _{x, 0} + j·Deg312 _{x, 1}
Vector_132 _x := Deg132 _{x, 0} + j·Deg132 _{x, 1}	Vector_313 _x := Deg313 _{x, 0} + j·Deg313 _{x, 1}
Vector_133 _x := Deg133 _{x, 0} + j·Deg133 _{x, 1}	Vector_314 _x := Deg314 _{x, 0} + j·Deg314 _{x, 1}
Vector_134 _x := Deg134 _{x, 0} + j·Deg134 _{x, 1}	Vector_315 _x := Deg315 _{x, 0} + j·Deg315 _{x, 1}
Vector_135 _x := Deg135 _{x, 0} + j·Deg135 _{x, 1}	Vector_316 _x := Deg316 _{x, 0} + j·Deg316 _{x, 1}
Vector_136 _x := Deg136 _{x, 0} + j·Deg136 _{x, 1}	Vector_317 _x := Deg317 _{x, 0} + j·Deg317 _{x, 1}
Vector_137 _x := Deg137 _{x, 0} + j·Deg137 _{x, 1}	Vector_318 _x := Deg318 _{x, 0} + j·Deg318 _{x, 1}
Vector_138 _x := Deg138 _{x, 0} + j·Deg138 _{x, 1}	Vector_319 _x := Deg319 _{x, 0} + j·Deg319 _{x, 1}
Vector_139 _x := Deg139 _{x, 0} + j·Deg139 _{x, 1}	Vector_320 _x := Deg320 _{x, 0} + j·Deg320 _{x, 1}
Vector_140 _x := Deg140 _{x, 0} + j·Deg140 _{x, 1}	Vector_321 _x := Deg321 _{x, 0} + j·Deg321 _{x, 1}
Vector_141 _x := Deg141 _{x, 0} + j·Deg141 _{x, 1}	Vector_322 _x := Deg322 _{x, 0} + j·Deg322 _{x, 1}
Vector_142 _x := Deg142 _{x, 0} + j·Deg142 _{x, 1}	Vector_323 _x := Deg323 _{x, 0} + j·Deg323 _{x, 1}
Vector_143 _x := Deg143 _{x, 0} + j·Deg143 _{x, 1}	Vector_324 _x := Deg324 _{x, 0} + j·Deg324 _{x, 1}
Vector_144 _x := Deg144 _{x, 0} + j·Deg144 _{x, 1}	Vector_325 _x := Deg325 _{x, 0} + j·Deg325 _{x, 1}
Vector_145 _x := Deg145 _{x, 0} + j·Deg145 _{x, 1}	Vector_326 _x := Deg326 _{x, 0} + j·Deg326 _{x, 1}
Vector_146 _x := Deg146 _{x, 0} + j·Deg146 _{x, 1}	Vector_327 _x := Deg327 _{x, 0} + j·Deg327 _{x, 1}
Vector_147 _x := Deg147 _{x, 0} + j·Deg147 _{x, 1}	Vector_328 _x := Deg328 _{x, 0} + j·Deg328 _{x, 1}
Vector_148 _x := Deg148 _{x, 0} + j·Deg148 _{x, 1}	Vector_329 _x := Deg329 _{x, 0} + j·Deg329 _{x, 1}
Vector_149 _x := Deg149 _{x, 0} + j·Deg149 _{x, 1}	Vector_330 _x := Deg330 _{x, 0} + j·Deg330 _{x, 1}
Vector_150 _x := Deg150 _{x, 0} + j·Deg150 _{x, 1}	Vector_331 _x := Deg331 _{x, 0} + j·Deg331 _{x, 1}
Vector_151 _x := Deg151 _{x, 0} + j·Deg151 _{x, 1}	Vector_332 _x := Deg332 _{x, 0} + j·Deg332 _{x, 1}
Vector_152 _x := Deg152 _{x, 0} + j·Deg152 _{x, 1}	Vector_333 _x := Deg333 _{x, 0} + j·Deg333 _{x, 1}
Vector_153 _x := Deg153 _{x, 0} + j·Deg153 _{x, 1}	Vector_334 _x := Deg334 _{x, 0} + j·Deg334 _{x, 1}
Vector_154 _x := Deg154 _{x, 0} + j·Deg154 _{x, 1}	Vector_335 _x := Deg335 _{x, 0} + j·Deg335 _{x, 1}
Vector_155 _x := Deg155 _{x, 0} + j·Deg155 _{x, 1}	Vector_336 _x := Deg336 _{x, 0} + j·Deg336 _{x, 1}

$\text{Vector}_{156}_x := \text{Deg}156_{x,0} + j \cdot \text{Deg}156_{x,1}$	$\text{Vector}_{337}_x := \text{Deg}337_{x,0} + j \cdot \text{Deg}337_{x,1}$
$\text{Vector}_{157}_x := \text{Deg}157_{x,0} + j \cdot \text{Deg}157_{x,1}$	$\text{Vector}_{338}_x := \text{Deg}338_{x,0} + j \cdot \text{Deg}338_{x,1}$
$\text{Vector}_{158}_x := \text{Deg}158_{x,0} + j \cdot \text{Deg}158_{x,1}$	$\text{Vector}_{339}_x := \text{Deg}339_{x,0} + j \cdot \text{Deg}339_{x,1}$
$\text{Vector}_{159}_x := \text{Deg}159_{x,0} + j \cdot \text{Deg}159_{x,1}$	$\text{Vector}_{340}_x := \text{Deg}340_{x,0} + j \cdot \text{Deg}340_{x,1}$
$\text{Vector}_{160}_x := \text{Deg}160_{x,0} + j \cdot \text{Deg}160_{x,1}$	$\text{Vector}_{341}_x := \text{Deg}341_{x,0} + j \cdot \text{Deg}341_{x,1}$
$\text{Vector}_{161}_x := \text{Deg}161_{x,0} + j \cdot \text{Deg}161_{x,1}$	$\text{Vector}_{342}_x := \text{Deg}342_{x,0} + j \cdot \text{Deg}342_{x,1}$
$\text{Vector}_{162}_x := \text{Deg}162_{x,0} + j \cdot \text{Deg}162_{x,1}$	$\text{Vector}_{343}_x := \text{Deg}343_{x,0} + j \cdot \text{Deg}343_{x,1}$
$\text{Vector}_{163}_x := \text{Deg}163_{x,0} + j \cdot \text{Deg}163_{x,1}$	$\text{Vector}_{344}_x := \text{Deg}344_{x,0} + j \cdot \text{Deg}344_{x,1}$
$\text{Vector}_{164}_x := \text{Deg}164_{x,0} + j \cdot \text{Deg}164_{x,1}$	$\text{Vector}_{345}_x := \text{Deg}345_{x,0} + j \cdot \text{Deg}345_{x,1}$
$\text{Vector}_{165}_x := \text{Deg}165_{x,0} + j \cdot \text{Deg}165_{x,1}$	$\text{Vector}_{346}_x := \text{Deg}346_{x,0} + j \cdot \text{Deg}346_{x,1}$
$\text{Vector}_{166}_x := \text{Deg}166_{x,0} + j \cdot \text{Deg}166_{x,1}$	$\text{Vector}_{347}_x := \text{Deg}347_{x,0} + j \cdot \text{Deg}347_{x,1}$
$\text{Vector}_{167}_x := \text{Deg}167_{x,0} + j \cdot \text{Deg}167_{x,1}$	$\text{Vector}_{348}_x := \text{Deg}348_{x,0} + j \cdot \text{Deg}348_{x,1}$
$\text{Vector}_{168}_x := \text{Deg}168_{x,0} + j \cdot \text{Deg}168_{x,1}$	$\text{Vector}_{349}_x := \text{Deg}349_{x,0} + j \cdot \text{Deg}349_{x,1}$
$\text{Vector}_{169}_x := \text{Deg}169_{x,0} + j \cdot \text{Deg}169_{x,1}$	$\text{Vector}_{350}_x := \text{Deg}350_{x,0} + j \cdot \text{Deg}350_{x,1}$
$\text{Vector}_{170}_x := \text{Deg}170_{x,0} + j \cdot \text{Deg}170_{x,1}$	$\text{Vector}_{351}_x := \text{Deg}351_{x,0} + j \cdot \text{Deg}351_{x,1}$
$\text{Vector}_{171}_x := \text{Deg}171_{x,0} + j \cdot \text{Deg}171_{x,1}$	$\text{Vector}_{352}_x := \text{Deg}352_{x,0} + j \cdot \text{Deg}352_{x,1}$
$\text{Vector}_{172}_x := \text{Deg}172_{x,0} + j \cdot \text{Deg}172_{x,1}$	$\text{Vector}_{353}_x := \text{Deg}353_{x,0} + j \cdot \text{Deg}353_{x,1}$
$\text{Vector}_{173}_x := \text{Deg}173_{x,0} + j \cdot \text{Deg}173_{x,1}$	$\text{Vector}_{354}_x := \text{Deg}354_{x,0} + j \cdot \text{Deg}354_{x,1}$
$\text{Vector}_{174}_x := \text{Deg}174_{x,0} + j \cdot \text{Deg}174_{x,1}$	$\text{Vector}_{355}_x := \text{Deg}355_{x,0} + j \cdot \text{Deg}355_{x,1}$
$\text{Vector}_{175}_x := \text{Deg}175_{x,0} + j \cdot \text{Deg}175_{x,1}$	$\text{Vector}_{356}_x := \text{Deg}356_{x,0} + j \cdot \text{Deg}356_{x,1}$
$\text{Vector}_{176}_x := \text{Deg}176_{x,0} + j \cdot \text{Deg}176_{x,1}$	$\text{Vector}_{357}_x := \text{Deg}357_{x,0} + j \cdot \text{Deg}357_{x,1}$
$\text{Vector}_{177}_x := \text{Deg}177_{x,0} + j \cdot \text{Deg}177_{x,1}$	$\text{Vector}_{358}_x := \text{Deg}358_{x,0} + j \cdot \text{Deg}358_{x,1}$
$\text{Vector}_{178}_x := \text{Deg}178_{x,0} + j \cdot \text{Deg}178_{x,1}$	$\text{Vector}_{359}_x := \text{Deg}359_{x,0} + j \cdot \text{Deg}359_{x,1}$
$\text{Vector}_{179}_x := \text{Deg}179_{x,0} + j \cdot \text{Deg}179_{x,1}$	
$\text{Vector}_{180}_x := \text{Deg}180_{x,0} + j \cdot \text{Deg}180_{x,1}$	

Phase shift and Amplitude for each element due to the transmitting angle:

$\alpha_0 := \text{Vector}_0$	$\alpha_{101} := \text{Vector}_{101}$	$\alpha_{201} := \text{Vector}_{201}$
$\alpha_1 := \text{Vector}_1$	$\alpha_{102} := \text{Vector}_{102}$	$\alpha_{202} := \text{Vector}_{202}$
$\alpha_2 := \text{Vector}_2$	$\alpha_{103} := \text{Vector}_{103}$	$\alpha_{203} := \text{Vector}_{203}$
$\alpha_3 := \text{Vector}_3$	$\alpha_{104} := \text{Vector}_{104}$	$\alpha_{204} := \text{Vector}_{204}$
$\alpha_4 := \text{Vector}_4$	$\alpha_{105} := \text{Vector}_{105}$	$\alpha_{205} := \text{Vector}_{205}$
$\alpha_5 := \text{Vector}_5$	$\alpha_{106} := \text{Vector}_{106}$	$\alpha_{206} := \text{Vector}_{206}$
$\alpha_6 := \text{Vector}_6$	$\alpha_{107} := \text{Vector}_{107}$	$\alpha_{207} := \text{Vector}_{207}$
$\alpha_7 := \text{Vector}_7$	$\alpha_{108} := \text{Vector}_{108}$	$\alpha_{208} := \text{Vector}_{208}$
$\alpha_8 := \text{Vector}_8$	$\alpha_{109} := \text{Vector}_{109}$	$\alpha_{209} := \text{Vector}_{209}$

$\alpha_{50} := \text{Vector}_{50}$	$\alpha_{151} := \text{Vector}_{151}$	$\alpha_{251} := \text{Vector}_{251}$
$\alpha_{51} := \text{Vector}_{51}$	$\alpha_{152} := \text{Vector}_{152}$	$\alpha_{252} := \text{Vector}_{252}$
$\alpha_{52} := \text{Vector}_{52}$	$\alpha_{153} := \text{Vector}_{153}$	$\alpha_{253} := \text{Vector}_{253}$
$\alpha_{53} := \text{Vector}_{53}$	$\alpha_{154} := \text{Vector}_{154}$	$\alpha_{254} := \text{Vector}_{254}$
$\alpha_{54} := \text{Vector}_{54}$	$\alpha_{155} := \text{Vector}_{155}$	$\alpha_{255} := \text{Vector}_{255}$
$\alpha_{55} := \text{Vector}_{55}$	$\alpha_{156} := \text{Vector}_{156}$	$\alpha_{256} := \text{Vector}_{256}$
$\alpha_{56} := \text{Vector}_{56}$	$\alpha_{157} := \text{Vector}_{157}$	$\alpha_{257} := \text{Vector}_{257}$
$\alpha_{57} := \text{Vector}_{57}$	$\alpha_{158} := \text{Vector}_{158}$	$\alpha_{258} := \text{Vector}_{258}$
$\alpha_{58} := \text{Vector}_{58}$	$\alpha_{159} := \text{Vector}_{159}$	$\alpha_{259} := \text{Vector}_{259}$
$\alpha_{59} := \text{Vector}_{59}$	$\alpha_{160} := \text{Vector}_{160}$	$\alpha_{260} := \text{Vector}_{260}$
$\alpha_{60} := \text{Vector}_{60}$	$\alpha_{161} := \text{Vector}_{161}$	$\alpha_{261} := \text{Vector}_{261}$
$\alpha_{61} := \text{Vector}_{61}$	$\alpha_{162} := \text{Vector}_{162}$	$\alpha_{262} := \text{Vector}_{262}$
$\alpha_{62} := \text{Vector}_{62}$	$\alpha_{163} := \text{Vector}_{163}$	$\alpha_{263} := \text{Vector}_{263}$
$\alpha_{63} := \text{Vector}_{63}$	$\alpha_{164} := \text{Vector}_{164}$	$\alpha_{264} := \text{Vector}_{264}$
$\alpha_{64} := \text{Vector}_{64}$	$\alpha_{165} := \text{Vector}_{165}$	$\alpha_{265} := \text{Vector}_{265}$
$\alpha_{65} := \text{Vector}_{65}$	$\alpha_{166} := \text{Vector}_{166}$	$\alpha_{266} := \text{Vector}_{266}$
$\alpha_{66} := \text{Vector}_{66}$	$\alpha_{167} := \text{Vector}_{167}$	$\alpha_{267} := \text{Vector}_{267}$
$\alpha_{67} := \text{Vector}_{67}$	$\alpha_{168} := \text{Vector}_{168}$	$\alpha_{268} := \text{Vector}_{268}$
$\alpha_{68} := \text{Vector}_{68}$	$\alpha_{169} := \text{Vector}_{169}$	$\alpha_{269} := \text{Vector}_{269}$
$\alpha_{69} := \text{Vector}_{69}$	$\alpha_{170} := \text{Vector}_{170}$	$\alpha_{270} := \text{Vector}_{270}$
$\alpha_{70} := \text{Vector}_{70}$	$\alpha_{171} := \text{Vector}_{171}$	$\alpha_{271} := \text{Vector}_{271}$
$\alpha_{71} := \text{Vector}_{71}$	$\alpha_{172} := \text{Vector}_{172}$	$\alpha_{272} := \text{Vector}_{272}$
$\alpha_{72} := \text{Vector}_{72}$	$\alpha_{173} := \text{Vector}_{173}$	$\alpha_{273} := \text{Vector}_{273}$
$\alpha_{73} := \text{Vector}_{73}$	$\alpha_{174} := \text{Vector}_{174}$	$\alpha_{274} := \text{Vector}_{274}$
$\alpha_{74} := \text{Vector}_{74}$	$\alpha_{175} := \text{Vector}_{175}$	$\alpha_{275} := \text{Vector}_{275}$
$\alpha_{75} := \text{Vector}_{75}$	$\alpha_{176} := \text{Vector}_{176}$	$\alpha_{276} := \text{Vector}_{276}$
$\alpha_{76} := \text{Vector}_{76}$	$\alpha_{177} := \text{Vector}_{177}$	$\alpha_{277} := \text{Vector}_{277}$
$\alpha_{77} := \text{Vector}_{77}$	$\alpha_{178} := \text{Vector}_{178}$	$\alpha_{278} := \text{Vector}_{278}$
$\alpha_{78} := \text{Vector}_{78}$	$\alpha_{179} := \text{Vector}_{179}$	$\alpha_{279} := \text{Vector}_{279}$
$\alpha_{79} := \text{Vector}_{79}$	$\alpha_{180} := \text{Vector}_{180}$	$\alpha_{280} := \text{Vector}_{280}$
$\alpha_{80} := \text{Vector}_{80}$	$\alpha_{181} := \text{Vector}_{181}$	$\alpha_{281} := \text{Vector}_{281}$
$\alpha_{81} := \text{Vector}_{81}$	$\alpha_{182} := \text{Vector}_{182}$	$\alpha_{282} := \text{Vector}_{282}$
$\alpha_{82} := \text{Vector}_{82}$	$\alpha_{183} := \text{Vector}_{183}$	$\alpha_{283} := \text{Vector}_{283}$
$\alpha_{83} := \text{Vector}_{83}$	$\alpha_{184} := \text{Vector}_{184}$	$\alpha_{284} := \text{Vector}_{284}$
$\alpha_{84} := \text{Vector}_{84}$	$\alpha_{185} := \text{Vector}_{185}$	$\alpha_{285} := \text{Vector}_{285}$
$\alpha_{85} := \text{Vector}_{85}$	$\alpha_{186} := \text{Vector}_{186}$	$\alpha_{286} := \text{Vector}_{286}$
$\alpha_{86} := \text{Vector}_{86}$	$\alpha_{187} := \text{Vector}_{187}$	$\alpha_{287} := \text{Vector}_{287}$
$\alpha_{87} := \text{Vector}_{87}$	$\alpha_{188} := \text{Vector}_{188}$	$\alpha_{288} := \text{Vector}_{288}$
$\alpha_{88} := \text{Vector}_{88}$	$\alpha_{189} := \text{Vector}_{189}$	$\alpha_{289} := \text{Vector}_{289}$
$\alpha_{89} := \text{Vector}_{89}$	$\alpha_{190} := \text{Vector}_{190}$	$\alpha_{290} := \text{Vector}_{290}$
$\alpha_{90} := \text{Vector}_{90}$	$\alpha_{191} := \text{Vector}_{191}$	$\alpha_{291} := \text{Vector}_{291}$

$\alpha_{91} := \text{Vector}_{91}$	$\alpha_{192} := \text{Vector}_{192}$	$\alpha_{292} := \text{Vector}_{292}$
$\alpha_{92} := \text{Vector}_{92}$	$\alpha_{193} := \text{Vector}_{193}$	$\alpha_{293} := \text{Vector}_{293}$
$\alpha_{93} := \text{Vector}_{93}$	$\alpha_{194} := \text{Vector}_{194}$	$\alpha_{294} := \text{Vector}_{294}$
$\alpha_{94} := \text{Vector}_{94}$	$\alpha_{195} := \text{Vector}_{195}$	$\alpha_{295} := \text{Vector}_{295}$
$\alpha_{95} := \text{Vector}_{95}$	$\alpha_{196} := \text{Vector}_{196}$	$\alpha_{296} := \text{Vector}_{296}$
$\alpha_{96} := \text{Vector}_{96}$	$\alpha_{197} := \text{Vector}_{197}$	$\alpha_{297} := \text{Vector}_{297}$
$\alpha_{97} := \text{Vector}_{97}$	$\alpha_{198} := \text{Vector}_{198}$	$\alpha_{298} := \text{Vector}_{298}$
$\alpha_{98} := \text{Vector}_{98}$	$\alpha_{199} := \text{Vector}_{199}$	$\alpha_{299} := \text{Vector}_{299}$
$\alpha_{99} := \text{Vector}_{99}$	$\alpha_{200} := \text{Vector}_{200}$	$\alpha_{300} := \text{Vector}_{300}$
$\alpha_{100} := \text{Vector}_{100}$		
$\alpha_{301} := \text{Vector}_{301}$	$\alpha_{331} := \text{Vector}_{31}$	
$\alpha_{302} := \text{Vector}_{302}$	$\alpha_{332} := \text{Vector}_{32}$	
$\alpha_{303} := \text{Vector}_{303}$	$\alpha_{333} := \text{Vector}_{333}$	
$\alpha_{304} := \text{Vector}_{304}$	$\alpha_{334} := \text{Vector}_{334}$	
$\alpha_{305} := \text{Vector}_{305}$	$\alpha_{335} := \text{Vector}_{335}$	
$\alpha_{306} := \text{Vector}_{306}$	$\alpha_{336} := \text{Vector}_{336}$	
$\alpha_{307} := \text{Vector}_{307}$	$\alpha_{337} := \text{Vector}_{337}$	
$\alpha_{308} := \text{Vector}_{308}$	$\alpha_{338} := \text{Vector}_{338}$	
$\alpha_{309} := \text{Vector}_{309}$	$\alpha_{339} := \text{Vector}_{339}$	
$\alpha_{310} := \text{Vector}_{310}$	$\alpha_{340} := \text{Vector}_{340}$	
$\alpha_{311} := \text{Vector}_{311}$	$\alpha_{341} := \text{Vector}_{341}$	
$\alpha_{312} := \text{Vector}_{312}$	$\alpha_{342} := \text{Vector}_{342}$	
$\alpha_{313} := \text{Vector}_{313}$	$\alpha_{343} := \text{Vector}_{343}$	
$\alpha_{314} := \text{Vector}_{314}$	$\alpha_{344} := \text{Vector}_{344}$	
$\alpha_{315} := \text{Vector}_{315}$	$\alpha_{345} := \text{Vector}_{345}$	
$\alpha_{316} := \text{Vector}_{16}$	$\alpha_{346} := \text{Vector}_{346}$	
$\alpha_{317} := \text{Vector}_{317}$	$\alpha_{347} := \text{Vector}_{347}$	
$\alpha_{318} := \text{Vector}_{318}$	$\alpha_{348} := \text{Vector}_{348}$	
$\alpha_{319} := \text{Vector}_{319}$	$\alpha_{349} := \text{Vector}_{349}$	
$\alpha_{320} := \text{Vector}_{320}$	$\alpha_{350} := \text{Vector}_{50}$	
$\alpha_{321} := \text{Vector}_{21}$	$\alpha_{351} := \text{Vector}_{351}$	
$\alpha_{322} := \text{Vector}_{322}$	$\alpha_{352} := \text{Vector}_{352}$	
$\alpha_{323} := \text{Vector}_{323}$	$\alpha_{353} := \text{Vector}_{353}$	
$\alpha_{324} := \text{Vector}_{324}$	$\alpha_{354} := \text{Vector}_{354}$	
$\alpha_{325} := \text{Vector}_{325}$	$\alpha_{355} := \text{Vector}_{355}$	
$\alpha_{326} := \text{Vector}_{26}$	$\alpha_{356} := \text{Vector}_{356}$	
$\alpha_{327} := \text{Vector}_{327}$	$\alpha_{357} := \text{Vector}_{357}$	
$\alpha_{328} := \text{Vector}_{328}$	$\alpha_{358} := \text{Vector}_{358}$	
$\alpha_{329} := \text{Vector}_{329}$	$\alpha_{359} := \text{Vector}_{359}$	
$\alpha_{330} := \text{Vector}_{330}$		

Frequencies Selections:

$$f_{ref} := 1.7$$

$$\lambda_{ref} := \frac{3 \cdot 10^8}{f_{ref} \cdot 10^9}$$

$$\lambda_{ref} = 0.176$$

$$d := \frac{\lambda_{ref}}{2}$$

$$d = 0.088$$

$$freq_{oper} := 1.7$$

$$\lambda := \frac{3 \cdot 10^8}{freq_{oper} \cdot 10^9}$$

$$\lambda = 0.176$$

$$k_{ref} := \frac{2 \cdot \pi}{\lambda_{ref}}$$

$$k_{ref} = 35.605$$

$$step_f := round\left(\frac{freq_{oper} - 0.8}{\Delta f}\right)$$

$$step_f = 106$$

$$k := \frac{2 \cdot \pi}{\lambda}$$

$$k = 35.605$$

Number of antenna elements:

$$N := 30$$

$$n := 0..N - 1$$

Transmitting direction:

$$\theta_{tx} := 10 \cdot \frac{\pi}{180}$$

$$\theta_{tx} = 0.175$$

Phase Reference

We will pick the furthest element as our reference element. The reference element has zero phase shift due to its location, because the position vector for this element is zero:

$$e^{j \frac{2 \cdot \pi}{\lambda} \cdot d \cdot n \cdot \sin(\theta_{tx})} = e^{j \frac{2 \cdot \pi}{\lambda} \cdot d \cdot 0 \cdot \sin(\theta_{tx})} = 1$$

Array Signals

Because of the array spatial distribution, there will be an additional phase shift associated with each element and each angle-of-transmission.

$$\Delta\psi := k_{ref} \cdot d \cdot \sin(\theta_{tx})$$

$$\Delta\psi := round\left(\Delta\psi \cdot \frac{180}{\pi}\right)$$

$$\Delta\psi = 31$$

There are 30 elements, thus there are 30 phasor values for the elements. Depending on the phase shift with respect to the reference element, the required phases for the elements are loaded into.

$$\begin{array}{lll}
x_0 := 0 & x_{10} := \text{mod}(x_1 + x_9, 360) & x_{20} := \text{mod}(x_1 + x_{19}, 360) \\
x_1 := \text{if}(\Delta\psi < 0, \Delta\psi + 360, \text{mod}(\Delta\psi, 360)) & x_{11} := \text{mod}(x_1 + x_{10}, 360) & x_{21} := \text{mod}(x_1 + x_{20}, 360) \\
x_2 := \text{mod}(x_1 + x_1, 360) & x_{12} := \text{mod}(x_1 + x_{11}, 360) & x_{22} := \text{mod}(x_1 + x_{21}, 360) \\
x_3 := \text{mod}(x_1 + x_2, 360) & x_{13} := \text{mod}(x_1 + x_{12}, 360) & x_{23} := \text{mod}(x_1 + x_{22}, 360) \\
x_4 := \text{mod}(x_1 + x_3, 360) & x_{14} := \text{mod}(x_1 + x_{13}, 360) & x_{24} := \text{mod}(x_1 + x_{23}, 360) \\
x_5 := \text{mod}(x_1 + x_4, 360) & x_{15} := \text{mod}(x_1 + x_{14}, 360) & x_{25} := \text{mod}(x_1 + x_{24}, 360) \\
x_6 := \text{mod}(x_1 + x_5, 360) & x_{16} := \text{mod}(x_1 + x_{15}, 360) & x_{26} := \text{mod}(x_1 + x_{25}, 360) \\
x_7 := \text{mod}(x_1 + x_6, 360) & x_{17} := \text{mod}(x_1 + x_{16}, 360) & x_{27} := \text{mod}(x_1 + x_{26}, 360) \\
x_8 := \text{mod}(x_1 + x_7, 360) & x_{18} := \text{mod}(x_1 + x_{17}, 360) & x_{28} := \text{mod}(x_1 + x_{27}, 360) \\
x_9 := \text{mod}(x_1 + x_8, 360) & x_{19} := \text{mod}(x_1 + x_{18}, 360) & x_{29} := \text{mod}(x_1 + x_{28}, 360)
\end{array}$$

$e^{j \cdot \Phi}$

component of the array factor

$$\Phi_{el_0} := \left(\arg(\alpha_{x_0}) \right)_{step_f}$$

$$\Phi_{el_1} := \left(\arg(\alpha_{x_1}) \right)_{step_f}$$

$$\Phi_{el_2} := \left(\arg(\alpha_{x_2}) \right)_{step_f}$$

$$\Phi_{el_3} := \left(\arg(\alpha_{x_3}) \right)_{step_f}$$

$$\Phi_{el_4} := \left(\arg(\alpha_{x_4}) \right)_{step_f}$$

$$\Phi_{el_5} := \left(\arg(\alpha_{x_5}) \right)_{step_f}$$

$$\Phi_{el_6} := \left(\arg(\alpha_{x_6}) \right)_{step_f}$$

$$\Phi_{el_7} := \left(\arg(\alpha_{x_7}) \right)_{step_f}$$

$$\Phi_{el_8} := \left(\arg(\alpha_{x_8}) \right)_{step_f}$$

$$\Phi_{el_9} := \left(\arg(\alpha_{x_9}) \right)_{step_f}$$

$$\Phi_{el_{10}} := \left(\arg(\alpha_{x_{10}}) \right)_{step_f}$$

$$\Phi_{el_{11}} := \left(\arg(\alpha_{x_{11}}) \right)_{step_f}$$

$$\Phi_{el_{12}} := \left(\arg(\alpha_{x_{12}}) \right)_{step_f}$$

$$\Phi_{el_{13}} := \left(\arg(\alpha_{x_{13}}) \right)_{step_f}$$

$$\Phi_{el_{14}} := \left(\arg(\alpha_{x_{14}}) \right)_{step_f}$$

$$\Phi_{el_{15}} := \left(\arg(\alpha_{x_{15}}) \right)_{step_f}$$

$$\Phi_{el_{16}} := \left(\arg(\alpha_{x_{16}}) \right)_{step_f}$$

$$\Phi_{el_{17}} := \left(\arg(\alpha_{x_{17}}) \right)_{step_f}$$

$$\Phi_{el_{18}} := \left(\arg(\alpha_{x_{18}}) \right)_{step_f}$$

$$\Phi_{el_{19}} := \left(\arg(\alpha_{x_{19}}) \right)_{step_f}$$

$$\Phi_{el_{20}} := \left(\arg(\alpha_{x_{20}}) \right)_{step_f}$$

$$\Phi_{el_{21}} := \left(\arg(\alpha_{x_{21}}) \right)_{step_f}$$

$$\Phi_{el_{22}} := \left(\arg(\alpha_{x_{22}}) \right)_{step_f}$$

$$\Phi_{el_{23}} := \left(\arg(\alpha_{x_{23}}) \right)_{step_f}$$

$$\Phi_{el_{24}} := \left(\arg(\alpha_{x_{24}}) \right)_{step_f}$$

$$\Phi_{el_{25}} := \left(\arg(\alpha_{x_{25}}) \right)_{step_f}$$

$$\Phi_{el_{26}} := \left(\arg(\alpha_{x_{26}}) \right)_{step_f}$$

$$\Phi_{el_{27}} := \left(\arg(\alpha_{x_{27}}) \right)_{step_f}$$

$$\Phi_{el_{28}} := \left(\arg(\alpha_{x_{28}}) \right)_{step_f}$$

$$\Phi_{el_{29}} := \left(\arg(\alpha_{x_{29}}) \right)_{step_f}$$

Amplitude read in based on the phase shift of the element with respect to the reference element.

$$A_0 := \left| \left(\alpha_{x_0} \right)_{step_f} \right|$$

$$A_1 := \left| \left(\alpha_{x_1} \right)_{step_f} \right|$$

$$A_{10} := \left| \left(\alpha_{x_{10}} \right)_{step_f} \right|$$

$$A_{11} := \left| \left(\alpha_{x_{11}} \right)_{step_f} \right|$$

$$A_{20} := \left| \left(\alpha_{x_{20}} \right)_{step_f} \right|$$

$$A_{21} := \left| \left(\alpha_{x_{21}} \right)_{step_f} \right|$$

$$\begin{aligned}
A_2 &:= \left| \left(\alpha_{x_2} \right)_{\text{step}_f} \right| & A_{12} &:= \left| \left(\alpha_{x_{12}} \right)_{\text{step}_f} \right| & A_{22} &:= \left| \left(\alpha_{x_{22}} \right)_{\text{step}_f} \right| \\
A_3 &:= \left| \left(\alpha_{x_3} \right)_{\text{step}_f} \right| & A_{13} &:= \left| \left(\alpha_{x_{13}} \right)_{\text{step}_f} \right| & A_{23} &:= \left| \left(\alpha_{x_{23}} \right)_{\text{step}_f} \right| \\
A_4 &:= \left| \left(\alpha_{x_4} \right)_{\text{step}_f} \right| & A_{14} &:= \left| \left(\alpha_{x_{14}} \right)_{\text{step}_f} \right| & A_{24} &:= \left| \left(\alpha_{x_{24}} \right)_{\text{step}_f} \right| \\
A_5 &:= \left| \left(\alpha_{x_5} \right)_{\text{step}_f} \right| & A_{15} &:= \left| \left(\alpha_{x_{15}} \right)_{\text{step}_f} \right| & A_{25} &:= \left| \left(\alpha_{x_{25}} \right)_{\text{step}_f} \right| \\
A_6 &:= \left| \left(\alpha_{x_6} \right)_{\text{step}_f} \right| & A_{16} &:= \left| \left(\alpha_{x_{16}} \right)_{\text{step}_f} \right| & A_{26} &:= \left| \left(\alpha_{x_{26}} \right)_{\text{step}_f} \right| \\
A_7 &:= \left| \left(\alpha_{x_7} \right)_{\text{step}_f} \right| & A_{17} &:= \left| \left(\alpha_{x_{17}} \right)_{\text{step}_f} \right| & A_{27} &:= \left| \left(\alpha_{x_{27}} \right)_{\text{step}_f} \right| \\
A_8 &:= \left| \left(\alpha_{x_8} \right)_{\text{step}_f} \right| & A_{18} &:= \left| \left(\alpha_{x_{18}} \right)_{\text{step}_f} \right| & A_{28} &:= \left| \left(\alpha_{x_{28}} \right)_{\text{step}_f} \right| \\
A_9 &:= \left| \left(\alpha_{x_9} \right)_{\text{step}_f} \right| & A_{19} &:= \left| \left(\alpha_{x_{19}} \right)_{\text{step}_f} \right| & A_{29} &:= \left| \left(\alpha_{x_{29}} \right)_{\text{step}_f} \right|
\end{aligned}$$

$$\Phi_{\text{el}}^T \cdot \frac{180}{\pi} = \begin{array}{|c|c|c|c|c|c|c|c|c|c|} \hline & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \hline 0 & -135.114 & -102.36 & -74.246 & -46.277 & -10.479 & 21.171 & 51.172 & 82.967 \\ \hline \end{array}$$

$$A^T = \begin{array}{|c|c|c|c|c|c|c|c|c|c|} \hline & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ \hline 0 & 0.411 & 0.41 & 0.444 & 0.404 & 0.387 & 0.429 & 0.408 & 0.424 & 0.45 & 0.386 \\ \hline \end{array}$$

Array Factor:

$$AF(\theta) := \sum_{n=0}^{N-1} A_n \cdot e^{j \cdot \Phi_{\text{el}} n \cdot e^{-j \cdot k \cdot d \cdot n \cdot \sin(\theta)}}$$

Plotting elevation radiation pattern:

Note that the referenced radiation pattern is plotted with operating frequency equals 1.7 GHz.

$$N_{\text{angles}} := 180$$

$$n := 0.. N_{\text{angles}} - 1$$

$$\Delta\theta := \frac{\pi}{N_{\text{angles}} - 1}$$

$$\Delta\theta = 0.018$$

$$\theta_{\text{obs},n} := n \cdot \Delta\theta - \frac{\pi}{2}$$

$$\text{mag_array_pattern}_n := 20 \cdot \log \left(\left(\left| AF(\theta_{\text{obs},n}) \right| \right) \right)$$

$$\text{max_val} := \text{max}(\text{mag_array_pattern})$$

$$\text{max_val} = 21.755$$

$$\text{max_plot} := \text{ceil}(\text{max}(\text{max_val}))$$

$$\text{max_plot} = 22$$

$$\theta_{\text{tx},\text{round}} := \Delta\theta \cdot \text{round} \left(\frac{\theta_{\text{tx}}}{\Delta\theta} \right)$$

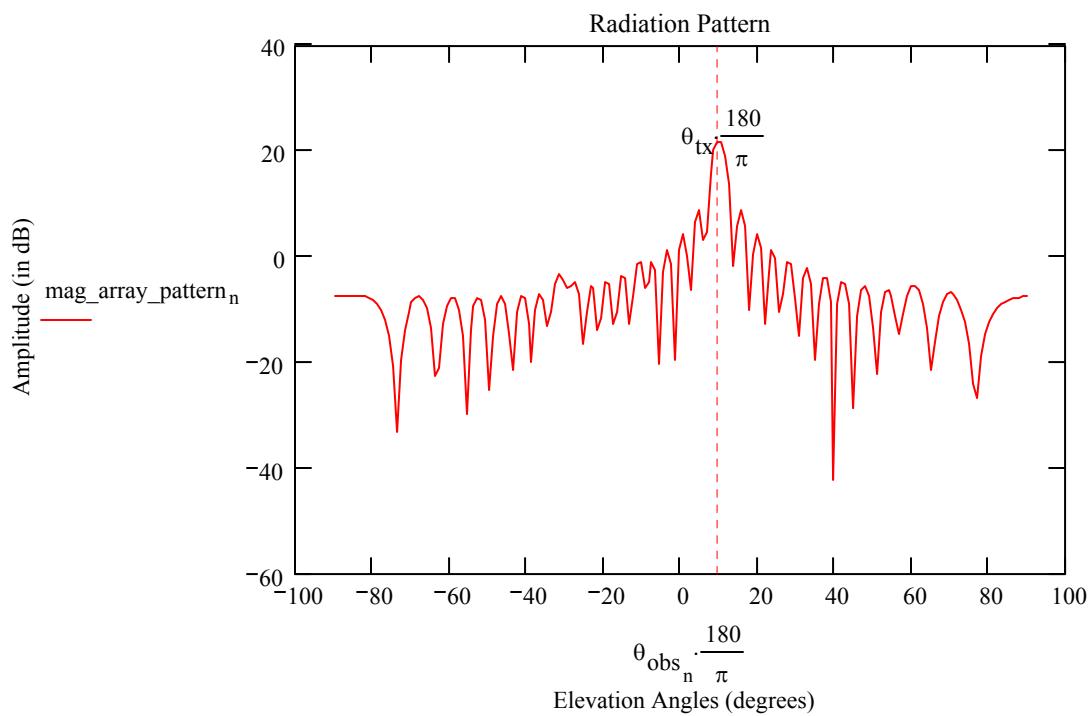
$$\theta_{tx_round} = 0.176$$

$$signal_n := \text{if}(\theta_{obs_n} = \theta_{tx_round}, \text{max_plot}, 0)$$

$$\theta_n := \left(\text{if}(mag_array_pattern_n = \text{max_val}, \theta_{obs_n}, 0) \cdot \frac{180}{\pi} \right)$$

$$\theta_{max} := \max(\theta)$$

$$\theta_{max} = 9.553$$



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APPENDIX D: MATLAB CODE FOR ARRAY RECEIVER

%This code is used to read in the measured I+, I-, Q+ and Q- from a data file “Demod_110102.m”.
% The single-ended I and Q components are converted to differential I and Q before computing the phases.

```

clear
load Demod_110103.m;
rad = pi/180;
degree = 180/pi;

phase_start = 0;
phase_stop = 355;
phase = [0:5:355];

%Read in IP, IN, QP & QN components.
IP = Demod_110103(:,1);
IN = Demod_110103(:,2);
QP = Demod_110103(:,3);
QN = Demod_110103(:,4);

%To get differential voltages.
Diff_I = (IP - IN);
Diff_Q = (QP - QN);

%Apply offset to the I & Q components.
Diff_I_off = Diff_I - 0.1;
Diff_Q_off = Diff_Q + 0.1;

%Measured phase, with and without offsets.
Phi = unwrap(atan2(Diff_Q,Diff_I))*(180/pi);
Phi_off = unwrap(atan2(Diff_Q_off,Diff_I_off));
Diff_I_max = max(Diff_I_off);
Diff_Q_max = max(Diff_Q_off);
A = Diff_I_max*cos(phase*rad);
B = Diff_Q_max*sin(phase*rad);

figure(1)
subplot(2,1,1)
plot(phase,Diff_I_off,phase,A,'x')
title('Transmitted Phase-In phase Response')
xlabel('Transmitted phase (degrees)'), ylabel('Measured differential In-Phase (volts)')
legend('Measured In-phase','Cosine')
subplot(2,1,2)
plot(phase,Diff_Q_off,phase,B,'x')
title('Transmitted Phase-Quadrature Response')
xlabel('Transmitted phase (degrees)'), ylabel('Measured differential Quadrature (volts)')
legend('Measured Quadrature','Sine')
print -depsc -tiff -r300 IQ

figure(2)
plot(phase,Phi)
title('Transmitted Phase-Measured Phase Response (w/o offsets)')
xlabel('Transmitted phase (degrees)'), ylabel('Measured phase (degrees)')
print -depsc -tiff -r300 phase_woff

%Theoretical result.
line = 1*phase + 100;
line0 = 1*phase + 0;

figure(3)

```

```

plot(phase,Phi_off*(180/pi),phase,line,'-')
title('Transmitted Phase-Measured Phase Resonse')
xlabel('Transmitted phase (degrees)'), ylabel('Measured phase (degrees)')
legend('Measured Phase','Theoretical Phase');
print -depsc -tiff -r300 phase_off

figure(4)
plot(phase,Phi_off*(180/pi)-100,phase,line0,'-')
title('Transmitted Phase-Measured Phase Resonse')
xlabel('Transmitted phase (degrees)'), ylabel('Measured phase (degrees)')
legend('Measured Phase','Theoretical Phase');
print -depsc -tiff -r300 phase_off_2

figure(5)
subplot(2,1,1)
plot(phase,Diff_I,phase,Diff_I_off);
title('In phase Component (without and with offset)')
xlabel('Transmitted phase (degrees)'), ylabel('Measured differential In-Phase (volts)')
legend('In-phase without offset','In-phase with offset')
subplot(2,1,2)
plot(phase,Diff_Q,phase,Diff_Q_off);
title('Quadrature Component (without and with offset)')
xlabel('Transmitted phase (degrees)'), ylabel('Measure differential Quadrature (volts)')
legend('Quadrature without offset','Quadrature with offset')
print -depsc -tiff -r300 IQ_2

```

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